# UNIVERSITÀ DEGLI STUDI DI MILANO 

Facoltà di Scienze e Tecnologie Corso di Laurea Magistrale in Fisica


# Manipulating ultracold atomic Fermi gases WITH TAILORED OPTICAL POTENTIALS 

Relatore interno: Prof. Davide Emilio Galli
Relatore esterno: Dott. Giacomo Roati
Correlatore: Dott. Francesco Scazza

Tesi di Laurea Magistrale di
Riccardo Panza
Matricola n. 896837
Codice P.A.C.S.: 03.75.-b

Questo è il fatto. E il fatto è la cosa più testarda del mondo.

Il Maestro e Margherita
Michail Afanas'evič Bulgakov

Quello che mi piace dell'esperienza è che si tratta di una cosa così onesta. Potete fare un mucchio di svolte sbagliate; ma tenete gli occhi aperti e non vi sarà permesso di spingervi troppo lontano prima che appaia il cartello giusto. Potete aver ingannato voi stessi, ma l'esperienza non sta ingannando voi. L'universo risponde il vero quando lo interrogate onestamente.

Sorpreso dalla gioia. I primi anni della mia vita
Clive Staples Lewis

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## Introduction

The ability to control and manipulate microscopic objects through laser beams is still proving to be a fertile field for new physics, as it has been recognized to Arthur Ashkin through the award of Nobel prize 2018. While optical tweezers are mostly applicable in capturing molecules and cells without harming them, the same concept of optical control has become routinely used in atomic physics to trap, cool and manipulate dilute samples, from single atoms and molecules to large ensambles of particles [1]. Indeed, two forces arise from the interaction between atoms and light: while the radiative force term is fundamental for all cooling techniques, trapping and manipulation methods rely on the dipole force, which is proportional to the light intensity profile. Local control of the intensity allows for addressing locally the atoms, both statically and dynamically.
Ultracold quantum gases offer a unique toolbox for quantum simulation of interacting many-body systems, thanks to high degree of controllability and the large tunability physical parameters [2, 3]. In the last decade, important tools were introduced in this framework, with the successful implementation of high-resolution detection of the atomic clouds and manipulation on single-atom length scales. These tools are currently applied in simulations of lattice-based quantum many-body systems, giving rise to the emerging field of quantum gas microscopy [4]. In addition, the flexibility in spatial light shaping offered by Digital Micromirrors Devices (DMD) and Spatial Light Modulators (SLM) finds natural applications in lattice quantum simulators [5] as well as in trapped quantum gases [6].
As shown in this work, the combined use of a high-resolution objective and a DMD in our experiment allowed us for manipulating a superfluid cloud of fermionic ${ }^{6} \mathrm{Li}$ atoms with tailored optical potentials, through the imprinting of a locally controlled phase: by illuminating the degenerate gas with different intensity profiles we were actually able to manipulate the phase of particles, also triggering collective dynamics. Furthermore, together with the capability of local control and manipulation of the gas afforded by DMD-tailored potentials, the two-dimensional confinement designed in this work opens new avenues for quantum simulations.
Morevoer, ultracold degenerate gases can precisely simulate systems in reduced dimensionality, such as two-dimensional fermionic systems: their peculiar and rich behaviour is a consequence of the interplay between statistics and dimensionality, which enhances the role of thermal fluctuations. The intriguing fundamental aspects in studying twodimensional systems are made even more interesting thanks to a large technological
impact: an example above all is the case of high- $T_{c}$ superconductors, which are still not completely understood theoretically, and represent a formidable challenge even for the most advanced computational techniques due to strong correlations between particles.
In this thesis work, I designed and characterized an optical set-up that generates a strongly anisotropic trapping beam, allowing to confine in a quasi-2D geometry a ${ }^{6} \mathrm{Li}$ Fermi gas. In particular, a phase-step plate is used to generate a tight well potential along the vertical axis of the beam. Moreover, after collaborating to the integration of the high-resolution objective and the DMD into the main experimental apparatus, I have also contributed to a series of experiments devoted to the control locally of the phase and the dynamics of an ultracold lithium cloud, through the application of spatially tailored potentials.
The outline of this thesis is the following: in the first chapter, I introduce the theoretical framework for light-atom interactions, with a particular focus on the optical dipole force. Then, I illustrate the basic features of degenerate Fermi gases, in the ideal non-interacting case, and of atomic scattering at ultralow temperatures, with particular emphasis on strongly interacting Fermi gases in the vicinity of a Feshbach resonance. In the last part of this chapter, I make an overview of our experimental set-up to cool and trap a degenerate ${ }^{6} \mathrm{Li}$ gas.
In the second chapter, I discuss the two fundamental tools I used to perform measurements in this work: a high-resolution objective and a Digital Micromirror Device (DMD), with a description of their technical properties and their implementation in the set-up. In the third chapter, I describe the phase imprinting experiments that we performed in our laboratory: first, i present a brief theoretical description of local phase manipulation, then I present and discuss the experimental results. The chapter ends with some perspectives about phase manipulation in ultracold gases.
In the fourth chapter, finally I describe the optical set-up for 2D confinement, after a brief theoretical introduction to the basic properties of 2D quantum gases. Then, the characterization of the quasi-2D confining beam is shown. The last part of this chapter deals with the optical schemes required to integrate the 2D confining potential into the existing experimental apparatus and apply them to the atomic samples.

## Chapter 1

## Trapped degenerate Fermi gases

Laser light is a powerful tool in the atomic physics field: not only lasers can be used to change the internal state of an atom, but they allow for manipulating external degrees of freedom through interaction between atoms and coherent light. This chapter is devoted to discuss the use of detuned laser light to produce non-uniform potentials in ultracold Fermi gases, with a particular insight on our system of ${ }^{6}$ Li degenerate gas. In the first part of this chapter, atom-light interaction is theoretically treated, with a particular focus on the conservative term. Then, in the second part of the chapter I'm presenting a brief theoretical introduction to ultracold Fermi gases. The third and last part gives an overview about our experimental set-up and routine to cool and trap the gas until degeneration.

### 1.1 Atoms in light fields

The effect of radiation on atoms was one of the first evidences of the quantum behaviour of microscopic systems, thanks to emission and absorption spectra. Phenomena of absorption and emission of resonant light are fundamental in atomic physics to provide the cooling and trapping effects at the beginning of development of the cold atoms field, but are only some of the possible effects: as Ashkin as shown in 1987 through his optical tweezers [7], other spectacular effects can occur, which are largely applicable in biophysics and biology, and are now routinary also in atomic physics. The model framework is the same I'm going to develop here, following [8]. After a discussion about some basic concepts of atom and light interaction, I concentrate on the optical dipole trap and the case of alkali atoms trapping.

### 1.1.1 Basic concepts of atom-light interaction

Although a complete theoretical model of interaction between atoms and light should be developed in a fully quantum framework, the semi-classical approach, which consider quantized atomic degrees of freedom and classical fields, catches all the typical features required to describe trapped ultracold gases. In this model, the presence of the field
induces a modification in the spatial charge distribution of the atom, causing a oscillating dipole at the driving frequency $\omega$; if the light frequency is close to the system resonant one $\omega_{0}$, it can also excite an oscillation of the charge distribution. Let us write the electric field in the usual complex notation as $\mathbf{E}(\mathbf{r}, t)=\hat{\mathbf{e}} \tilde{E}(\mathbf{r}) \exp (-i \omega t)+c . c$. and the induced dipole as $\mathbf{p}(\mathbf{r}, t)=\hat{\mathbf{e}} \tilde{p}(\mathbf{r}) \exp (-i \omega t)+c . c$. , where $\hat{\mathbf{e}}$ is the unit polarization vector. The complex amplitudes $\tilde{E}$ and $\tilde{p}$ are simply related by

$$
\begin{equation*}
\tilde{p}=\alpha \tilde{E} \tag{1.1}
\end{equation*}
$$

where $\alpha$ is the complex AC polarizability, which depends on the frequency $\omega$.
The interaction between the field and an atom, in this framework, is due to the real and imaginary part of the product between the induced dipole $\mathbf{p}$ and the electric field $\mathbf{E}$. The real part corresponds to the interaction potential of a dipole in a field:

$$
\begin{equation*}
U_{d i p}=-\frac{1}{2}\langle\mathbf{p} \mathbf{E}\rangle=-\frac{1}{2 \epsilon_{0} c} \Re[\alpha] I \tag{1.2}
\end{equation*}
$$

where the angular brackets denote the time average over the fast oscillating terms, $I=2 \epsilon_{0} c|\tilde{E}|^{2}$ is the field intensity and the factor $1 / 2$ takes into account the dipole is not permanent but induced. The potential felt by an atom is thus proportional to the intensity profile through the real part of the polarizability and represents the in-phase induced dipole oscillation, responsible of the dispersive properties of the interaction.
From Eq. 1.2 we can compute the force applied on the atom, the so-called dipole force, applying the gradient:

$$
\begin{equation*}
\mathbf{F}_{d i p}(\mathbf{r})=-\nabla U_{d i p}(\mathbf{r})=\frac{1}{2 \epsilon_{0} c} \Re[\alpha] \nabla I(\mathbf{r}) \tag{1.3}
\end{equation*}
$$

The dipole force is thus a conservative force, proportional to the intensity gradient of the driving field.
The imaginary part of the product between the induced dipole and the electric field is on the other hand connected to absorption effects, and thus to a dissipative term. We can indeed write the power absorbed by the atom from the driving field as:

$$
\begin{equation*}
P_{a b s}=\langle\dot{\mathbf{p}} \mathbf{E}\rangle=2 \omega \Im\left[\tilde{p} \tilde{E}^{\star}\right]=\frac{\omega}{\epsilon_{0} c} \Im[\alpha] I(\mathbf{r}) \tag{1.4}
\end{equation*}
$$

The absorption results from the imaginary part of the polarizability, which is connected to the out-of-phase oscillation of the induced dipole (so, it's connected to incoherent processes) ${ }^{1}$. If we consider the light as a stream of photons with energy $\hbar \omega$, the dissipative term of atom interaction can be interpreted as photon-scattering, in cycles of absorption and consequent spontaneous re-emission processes. In this way, we can compute the corresponding scattering rate:

$$
\begin{equation*}
\Gamma_{s c}(\mathbf{r})=\frac{P_{a b s}}{\hbar \omega}=\frac{1}{\hbar \epsilon_{0} c} \Im[\alpha] I(\mathbf{r}) \tag{1.5}
\end{equation*}
$$

[^0]Also this term gives rise to a force acting on the atoms, the radiative force: for resonant light, the absorption of a photon causes an atom to recoil, with a momentum transfer of $\hbar \mathbf{k}, \mathbf{k}$ being the wave-vector of the radiation, whereas the spontaneous emission causes a recoil in a random homogeneously-distributed direction. Thus, the atom scatters the photons and, after many photon-scattering events, the re-emission recoils average to zero, while the absorption ones sum together to give rise to the radiative force:

$$
\begin{equation*}
F_{r a d}=\hbar \mathbf{k} \Gamma_{s c} \tag{1.6}
\end{equation*}
$$

### 1.1.2 The atomic polarizability

The atomic polarizability $\alpha$ is a key quantity: in the two-level approximation, the atom can be considered as a two-level system, with only ground (g) and excited (e) states ${ }^{2}$ The frequency $\omega$ of the driving field is a fundamental variable for this kind of interactions: if $\omega$ is close to the resonance frequency $\omega_{0}$ of the two-levels system, the dissipative term is dominant and it is used in systems like optical molasses for laser cooling techniques (for a detailed treatment see, among the other textbooks, ( $\sqrt[9]{9})$. In this situation, the quantum effects like strongly populated excited state are playing a critical role in the calculations of $\alpha$.
To avoid the dissipative effects to limit in lifetime or to heat the sample, the conservative term of interaction has to be made dominant: this condition can be assured considering off-resonant light. In this condition, we can compute much more simply the polarizability: in case of negligible saturation effects, indeed, the calculation performed in the semiclassical two-levels model leads to the same results of the classical calculations of the Lorentz's model [10]:

$$
\begin{equation*}
\alpha=6 \pi \epsilon_{0} c^{3} \frac{\Gamma / \omega_{0}^{2}}{\omega_{0}^{2}-\omega^{2}-i\left(\omega^{3} / \omega_{0}^{2}\right) \Gamma} \tag{1.7}
\end{equation*}
$$

where $\Gamma$ is the damping rate, which corresponds to the spontaneous decay rate of the excited level, can be computed as function of the dipole $\hat{\mu}$ matrix element between excited and ground state as

$$
\begin{equation*}
\left.\Gamma=\frac{\omega_{0}^{3}}{3 \pi \epsilon_{0} \hbar c^{3}}|\langle e| \hat{\mu}| g\right\rangle\left.\right|^{2} \tag{1.8}
\end{equation*}
$$

Using this expression for the polarizability in the expressions Eq. (1.2) and Eq. (1.5), we can compute the dipole force and scattering rate. Assuming negligible saturation and large detunings, the expression can be approximated as:

$$
\begin{align*}
U_{d i p} & =-\frac{3 \pi c^{2}}{2 \omega_{0}^{3}}\left(\frac{\Gamma}{\omega_{0}-\omega}+\frac{\Gamma}{\omega_{0}+\omega}\right) I(\mathbf{r}) \\
\Gamma_{s c} & =\frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}}\left(\frac{\omega}{\omega_{0}}\right)^{3}\left(\frac{\Gamma}{\omega_{0}-\omega}+\frac{\Gamma}{\omega_{0}+\omega}\right)^{2} I(\mathbf{r}) \tag{1.9}
\end{align*}
$$

[^1]If the relation $|\Delta|=\left|\omega-\omega_{0}\right| \ll \omega+\omega_{0}$ is valid, we can apply the rotating-wave approximation (RWA) to Eq. (1.9), neglecting the term proportional to $1 /\left(\omega_{0}+\omega\right.$ and approximating $\omega / \omega_{0} \simeq 1$, and we obtain the relations

$$
\begin{align*}
U_{d i p} & =\frac{3 \pi c^{2}}{2 \omega_{0}^{3}}\left(\frac{\Gamma}{\Delta}\right) I(\mathbf{r})  \tag{1.10}\\
\Gamma_{s c} & =\frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}}\left(\frac{\Gamma}{\Delta}\right)^{2} I(\mathbf{r})
\end{align*}
$$

from which we obtain the very interesting relation:

$$
\begin{equation*}
\hbar \Gamma_{s c}=\frac{\Gamma}{\Delta} U_{d i p} \tag{1.11}
\end{equation*}
$$

Eq. (1.10) and Eq. (1.11) show us two important features of the optical dipole trapping:

- the sign of the detuning: for frequencies below atomic resonance, the laser is said to be "red-detuned" and $\Delta<0$; the dipole potential is thus negative, the atoms are attracted in light field and intensity maxima correspond to equilibrium points of the dipole potential. For frequencies above atomic resonance, on the other hand, the laser is said to be "blue-detuned" and $\Delta>0$; thus, the dipole interaction repels the atoms out of the field, and light field maxima correspond to maxima of the dipole potential too;
- the different scaling for dipole potential and scattering rate, because $U_{\text {dip }} \propto I / \Delta$ while $\Gamma_{s c} \propto I / \Delta^{2}$, allow to reduce the dissipative effects due to photon scattering without affecting the trap depth: indeed, increasing both detuning and intensity of the field, it's possible to reduce scattering rate with the same dipole potential.

However, for we are trapping and manipulating ${ }^{6} \mathrm{Li}$ atoms with largely detuned lasers from the dominant dipole allowed transitions $D_{1}$ and $D_{2}$ (see Fig. 1.2), the RWA is not applicable and the two terms in Eq. (1.9) are comparable: in this case, the first comment


Figure 1.1: Sketch of the frequency detuning on the optical dipole potential: the red-detuned beam (red trace) gives rise to an attractive potential for the atoms, while the blue-detuned beam (blue trace) gives rise a repulsive potential.
is still valid because for positive frequencies $\left|\omega_{0}-\omega\right|<\omega_{0}+\omega$, whereas the second one doesn't hold anymore. Anyway, low scattering rate are still achievable, as discussed in [11.

### 1.1.3 Multi-level atoms

The ideal case studied before is an interesting insight of the fundamental properties of the optical potential, but cannot take in account the more complex structure of multilevel atoms: the main consequence of the atom level structure is the dependence of the dipole potential on the particular sub-state of the atom. This leads to some quantitative modifications and also interesting new effects.
There are different ways to treat the problem in case of multi-level atoms: the polarizability can be considered state-dependent, or the ground-state potential can be statedependent. In this last case, the presence of a Hamiltonian due to a far-detuned electric field is treated as a second-order time-independent perturbation to non-degenerate states (i.e. the perturbation is linear in the intensity); for a perturbation $\mathcal{H}_{1}$, the energy shift $\Delta E_{i}$ for the unperturbed $i$-th state is given by

$$
\begin{equation*}
\Delta E_{i}=\sum_{j \neq i} \frac{\left.\left|\langle j| \mathcal{H}_{1}\right| i\right\rangle\left.\right|^{2}}{\mathcal{E}_{j}-\mathcal{E}_{i}} \tag{1.12}
\end{equation*}
$$

In our case, the perturbation Hamiltonian is due to the interaction between atom and light (light shift), and can be written as $\mathcal{H}_{1}=-\hat{\mu} \mathbf{E}$, where $-\hat{\mu}=-e-\hat{\mathbf{r}}$ is the electric dipole operator. Using this definition in Eq. (1.12), after some calculations, we get to the main result of the energy shift of one ground state $\left|g_{i}\right\rangle[8]$ :

$$
\begin{equation*}
\Delta E_{i}=\frac{3 \pi c^{2} \Gamma}{2 \omega_{0, i}^{3}} I \sum_{j} c_{i j}^{2}\left(\frac{1}{\omega_{0, i}-\omega_{j}}+\frac{1}{\omega_{0, i}+\omega_{j}}\right) \tag{1.13}
\end{equation*}
$$

where the summation is carried out over all the electronic excited states $\left|e_{j}\right\rangle$, and $c_{i j}$ is a transition coefficient related to the coupling strength between the $i$-th and $j$-th states, and is depending on the laser polarization and the electronic and angular momentum involved. As a consequence, to calculate the state-dependent ground-state dipole potential $U_{d i p, i}=$ $\Delta E_{i}$, one has to sum up all the contributions of the coupled excited states, taking into account relevant line strengths $c_{i j}^{2}$ and detunings $\Delta_{i j}$.
In particular, the case we are interested in here is the one of alkali atoms: in Fig. 1.2 is shown the structure of the electronic lowest levels of the ${ }^{6} \mathrm{Li}$, which is the element used in our experiment, but a similar structure is shared within all the alkali atoms. The case of alkali atoms is important because, since they have only one electron in the external shell, their excitation spectrum is simple and they have closed optical transitions lying in a convenient spectral range. The energy scales we need to consider in the discussion are the fine-structure splitting of the excited state $\hbar \Delta_{F S}^{\prime}$, and the hyperfine-structure splittings of the ground and excited state $\hbar \Delta_{H F S}$ and $\hbar \Delta_{H F S}^{\prime}$; all this three are relevant because $\hbar \Delta_{F S}^{\prime} \gg \hbar \Delta_{H F S} \gg \hbar \Delta_{H F S}^{\prime}$.


Figure 1.2: Structure of $D_{1}$ and $D_{2}$ lines of ${ }^{6} \mathrm{Li}$, with fine and hyperfine splittings. Figure taken from 12

Depending on the detuning from the transition frequency, in the optical dipole potential it could be necessary to take account of the fine-structure or hyperfine-structure splitting. The usual case, and the one we're going to study here, is the one in which the detuning scale is much higher than the hyperfine corrections to the energy but comparable with the fine correction of the excited state. In this case, the only relevant terms are the ones related to fine structure, while the hyperfine structure is not resolved and can be thus neglected: in this case, the different hyperfine states are degenerate.
Having $J=1 / 2,3 / 2$ in the excited state, and $I=1$ for a nucleus of ${ }^{6} \mathrm{Li}$, we can compute the number of available excited states: in the $2{ }^{2} P_{1 / 2}$ ( $D_{1}$ line), there are two states available, whereas in the $2{ }^{2} P_{3 / 2}$ ( $D_{2}$ line), the available states are four. For linear polarized light ${ }^{3}$, this corresponds only to a double strength of the $D_{2}$ line. The dipole potential takes thus the form

$$
\begin{align*}
U_{d i p}(\mathbf{r})=- & {\left[\frac{\pi c^{2} \Gamma}{2 \omega_{D_{1}}^{3}}\left(\frac{1}{\omega_{D_{1}}-\omega}+\frac{1}{\omega_{D_{1}}+\omega}\right)\right.} \\
& \left.+\frac{\pi c^{2} \Gamma}{\omega_{D_{2}}^{3}}\left(\frac{1}{\omega_{D_{2}}-\omega}+\frac{1}{\omega_{D_{2}}+\omega}\right)\right] I(\mathbf{r}) \tag{1.14}
\end{align*}
$$

because $\Gamma_{D 1}=\Gamma_{D 2}=\Gamma=5.8724 \mathrm{MHz}$ for Lithium $6[12]$. This was the expression of the dipole trap I used to study the quasi-2D confining beam (see Chap. 4): in my case, I actually didn't consider any other level contribution, as it should be done when the RWA isn't valid [9], because of the linewidth of the higher energy transitions of Li is much lower e.g. the one for the closest line at 323 nm is about $1 / 10$ less [13].
The two main lights used for optical traps in our experiment are a green 532 nm source, which is blue-detuned and thus generating repulsive potential, and infrared 1064 nm sources, which are red-detuned and thus generating confining potentials.
For the photon-scattering rate $\Gamma_{s c}$ in multi-level atoms, the same line strength factors used before are relevant, because both absorption and light shifts are determined by the same transition matrix elements. Thus, in the same conditions for the Eq. (1.14), we obtain the scattering rate

$$
\begin{align*}
\Gamma_{s c}(\mathbf{r})= & {\left[\frac{\pi c^{2} \Gamma^{2}}{2 \hbar \omega_{D_{1}}^{3}}\left(\frac{\omega}{\omega_{D_{1}}}\right)^{3}\left(\frac{1}{\omega_{D_{1}}-\omega}+\frac{1}{\omega_{D_{1}}+\omega}\right)^{2}\right.} \\
& \left.+\frac{\pi c^{2} \Gamma^{2}}{\hbar \omega_{D_{2}}^{3}}\left(\frac{\omega}{\omega_{D_{2}}}\right)^{3}\left(\frac{1}{\omega_{D_{2}}-\omega}+\frac{1}{\omega_{D_{2}}+\omega}\right)^{2}\right] I(\mathbf{r}) \tag{1.15}
\end{align*}
$$

### 1.2 Degenerate Fermi gases

In this section, I'm presenting the main properties of atomic Fermi gases, in particular in harmonic traps. Since a harmonic trap for an ultracold gas can be realized by the use of optical dipole trap and let us to achieve quantum degeneracy, as I explain in the

[^2]third part of the chapter, the discussion about the basic properties of Fermi gases in this condition represent the link to the discussion of the previous section.

### 1.2.1 Quantum statistics

At room temperature, the behaviour of a gas is described with classical statistical methods, while, when decreasing the temperature, they are expected to show more peculiar behaviours: indeed, at low temperatures, the de Broglie wavelength

$$
\begin{equation*}
\lambda_{d B}=\sqrt{\frac{2 \pi \hbar^{2}}{m k_{B} T}} \tag{1.16}
\end{equation*}
$$

which represents the extent of the wavepacket associated to a particle, becomes comparable with the mean inter-particle spacing $n^{-1 / 3}, n$ being the particle density. When this condition is reached, single-particle wavefunctions begin to overlap, the phase-space density $n \times \lambda_{d B}^{3}$ reaches the value of 1 , and particles trajectories aren't well defined anymore, making two identical particles no more distinguishable. Thus quantum collective behaviours are emerging in the sample.
The specific properties of this degenerate systems depends on the kind of particle involved and the consequent symmetry of the wavefunction: if under exchange of particles, the wavefuction is symmetric, the particles are bosons, while if it's antisymmetric, they are fermions. Quantum statistics is the fundamental tool to describe an ensamble of identical quantum particles.
The strong difference between bosons and fermions is already evident even considering only two particles: in this case, the wavefunction of the system depends on the coordinates of both of them and can be written in general as

$$
\begin{equation*}
\left\langle\zeta_{1}, \zeta_{2} \mid S \psi\right\rangle=\psi\left(\zeta_{1}, \zeta_{2}\right) \tag{1.17}
\end{equation*}
$$

Now, we can apply the exchange operator $\mathcal{P}_{12}$ on the state: the indistinguishability between particles means no measurement can be performed to detect the exchange; thus, the state obtained by the exchange of the two particles must be physically identical to the previous one, i.e. the wavefunctions can differ only by an arbitrary phase $\phi_{4}^{4}$

$$
\begin{equation*}
\mathcal{P}_{12} \psi\left(\zeta_{1}, \zeta_{2}\right)=\psi\left(\zeta_{2}, \zeta_{1}\right)=e^{i \phi} \psi\left(\zeta_{1}, \zeta_{2}\right) \tag{1.18}
\end{equation*}
$$

If we apply the exchange operator again to the state, we obtain the initial state, but the function $\psi$ gains another term of phase, becoming $e^{2 i \phi}$. Thus, $e^{2 i \phi}=1$ : this can only mean that $e^{i \phi}= \pm 1$. This provides that bosons, which have symmetric wavefucntion, can stay all in the same energy state, while not even two fermions, which have antisymmetric wavefunction, can be in the same state (Fermi exclusion principle).
The striking consequence of these features is very different between bosons and fermions: the bosons under the critical temperature $T_{c}$ are populating massively the ground state,

[^3]

Figure 1.3: Difference in level populations between bosons and fermions in the example of a harmonic potential.
giving rise to a macroscopic quantum object called Bose-Einstein Condensate (BEC); the fermions instead, at temperature lower than the Fermi temperature $T_{F}$, are filling up all the available states from the lowest state to an energy $E_{F}$.
The distribution functions, corresponding to the mean occupation number of state with energy $\epsilon$, for both bosons and fermions can be written together in the form

$$
\begin{equation*}
f \epsilon=\frac{1}{e^{\beta(\epsilon-\mu)} \pm 1} \tag{1.19}
\end{equation*}
$$

where $\epsilon$ in the energy of a level, $\mu$ the chemical potential of the gas, fixed by total atoms number $N$, and $\beta=1 / k_{B} T$. The bosons are characterized by the minus sign (Bose-Einstein statistics), which allows the macroscopic population of the ground state, whereas the fermions are characterized by the plus sign (Fermi-Dirac statistics), which instead tells us that mean occupation number cannot be higher than one (see Fig. 1.3). Atoms are composite particles, and their statistic is determined by the parity of the number of elementary fermions composing them: exchanging a couple of composite identical particles, indeed, corresponds to exchange multiple elementary identical particles pairs. Thus, if we have odd number of elementary fermions, the atoms are obeying to the FermiDirac statistics, whereas for a even number atoms are obeying to the Bose-Einstein one.

However, the quantum degeneracy cannot occur in a gaseous sample at usual temperature and density, as the thermodynamics conditions required would cause in the sample a transition to solid state, the only exception being Helium. As a consequence, a way to achieve degeneracy is the out of equilibrium condition of dilute regime, $n<10^{15} \mathrm{~cm}^{-3}$. Indeed, while solid state transition needs a certain amount of time, to let three-body collisions to allow the formation of bonds between atom 5 the thermalization of the gas requires only two-body collisions. But the two-body collisions rate is proportional to density, while the three-body one scales with density squared: thus, for sufficiently

[^4]low densities is possible to have the gas to enter a regime in which three-body collisions are strongly suppressed and the lifetime of quantum degenerate gas is extended, keeping thermal equilibrium still reachable.

### 1.2.2 Fermi gases in harmonic traps

The energy variable in the Fermi-Dirac distribution (see Fig. 1.4) is composed by the kinetic contribution and the potential energy, $\epsilon=p^{2} /(2 m)+V(\mathbf{r})$. Experiments with atomic cold gases are usually performed in optical traps, which can be well approximated to a harmonic potential at ultralow temperatures. The $d$-dimensional trapping potential can thus be written as

$$
\begin{equation*}
V\left(x_{1}, \ldots, x_{d}\right)=\frac{1}{2} m \sum_{i=1}^{d} \omega_{i}^{2} x_{i}^{2} \tag{1.20}
\end{equation*}
$$

where $m$ is the atomic mass and $\omega_{i}$ is the $i$-th trapping frequency. The density of states in a $d$-dimensional harmonic trap is defined by

$$
\begin{equation*}
g(\epsilon)=g_{s} \frac{\epsilon^{d-1}}{(d-1)!\Pi_{i=1}^{d} \hbar \omega_{i}} \tag{1.21}
\end{equation*}
$$

where $g_{s}=2 s+1$ is the number of degenerate levels of atoms with spin $s$. In 3 dimensions we obtain

$$
\begin{equation*}
g(\epsilon)=g_{s} \frac{\epsilon^{2}}{2(\hbar \bar{\omega})^{3}} \tag{1.22}
\end{equation*}
$$

where $\bar{\omega}=\left(\omega_{x} \omega_{y} \omega_{z}\right)^{1 / 3}$ is the geometric mean of the trapping frequencies. The Fermi energy can be computed trough the relation

$$
\begin{equation*}
N=\int_{0}^{\infty} g(\epsilon) f(\epsilon) d \epsilon \tag{1.23}
\end{equation*}
$$

Using the Fermi-Dirac distribution, Eq. 1.19), at $T=0$, defining $\mu(T=0)=E_{F}$ we obtain the relation

$$
\begin{equation*}
N=\int_{0}^{\infty} g(\epsilon) \Theta\left(\epsilon-E_{F}\right) d \epsilon \tag{1.24}
\end{equation*}
$$

where $\Theta$ is the Heaviside theta function. Fixing the number of atoms $N$, we can compute the Fermi energy:

$$
\begin{equation*}
E_{F}=(6 N)^{1 / 3} \hbar \bar{\omega} \tag{1.25}
\end{equation*}
$$

The Fermi energy sets the most important scale of energy in the system, and can be related to the gas peak density in the center of the trap $n(0)$ via $E_{F}=\hbar^{2} k_{F}^{2} /(2 m)=$ $\left(\hbar^{2} / 2 m\right)\left(6 \pi^{2} n(0)\right)^{2 / 3}$. The chemical potential at finite temperature, due to inhomogeneity of the harmonic potential, becomes position dependent, and can be computed in the local-density approximation (LDA), which approximate the gas as locally uniform

$$
\begin{equation*}
\mu(\mathbf{r})=\mu-V(\mathbf{r}) \tag{1.26}
\end{equation*}
$$



Figure 1.4: Fermi distribution profile for different temperatures under the Fermi degeneracy temperature: decreasing the ratio between $T$ and $T_{F}$, the distribution approaches to the Heaviside theta distribution.
with $V(\mathbf{r})$ being the trapping potential. The dependence on position of the chemical potential is adding many features to the behaviour of the gas [14]: among them, it's useful to underline that the trap is giving a time reference for comparing the dynamics of collective modes through the trap frequencies.
Integrating the distribution function in momentum space in the semiclassical approximation [15], it's possible to compute the density profile function:

$$
\begin{equation*}
n_{F}(\mathbf{r}, T=0)=\int \frac{d \mathbf{p}}{(2 \pi)^{3}} \frac{1}{e^{\beta\left(\frac{\mathbf{p}^{2}}{2 m}-V(\mathbf{r})-\mu\right)}+1}=-\frac{1}{\lambda_{d B}^{3}} \mathrm{Li}_{3 / 2}\left(-e^{\beta[\mu-V(\mathbf{r})]}\right) \tag{1.27}
\end{equation*}
$$

where $\mathrm{Li}_{n}$ is the $n$-th order polylogarithmic function [16].
At zero temperature, the density distribution Eq. 1.27) has a polynomial expression proportional to the term $\left[1-\left(x_{i} / R_{i, F}\right)^{2}\right]^{3 / 2}$, vanishing at the distance

$$
\begin{equation*}
R_{i, F}=\sqrt{\frac{E_{F}}{m \omega_{i}^{2}}}=(48 N)^{1 / 6} \sqrt{\frac{\hbar}{m \bar{\omega}}} \frac{\bar{\omega}}{\omega_{i}} \tag{1.28}
\end{equation*}
$$

called Thomas-Fermi radius. This quantity defines the cloud dimensions at zero temperature. It's interesting to compare this result to the one of the non-interacting Bose gas $R_{i, F}=a_{i, H O}=\sqrt{\hbar / m \omega_{i}}$ : the term proportional to number of particles $N$ in Eq. (1.28)
is a consequence of the Fermi gas additional pressure due to Pauli exclusion principle, which makes the fermionic cloud much more extended than the bosonic one.

### 1.2.3 Scattering theory of ultracold collisions

All the features described so far are good approximations for non-interacting or weakly interacting Fermi gases. Here, I present the collisional properties of an ultracold gas, and the connection between this properties and the interactions in the gas.

## Elastic collisions

The collisions in a cold atomic gases can be simply understood thanks to some properties of the atomic interactions: firstly, the interaction are of van der Waals type, $\propto-\frac{C_{6}}{r^{6}}$, where $C_{6}$ is the van der Waals coefficient; this kind of interaction are short-ranged i.e. beyond a certain distance $r_{v d W}$ the interaction is becoming negligible, and isotropic i.e. centralfield like. Furthermore, at very short distance (about few $a_{0}$, where the two electronic clouds are overlapping), two atoms are strongly repelling each other, due to Coulomb interactions and Pauli exclusion principle effect.
Secondly, the dilute regime requested to avoid three-body collisions and thus the solid formation can be expressed via the relation between density and van der Waals range

$$
\begin{equation*}
n r_{v d W}^{3} \ll 1 \tag{1.29}
\end{equation*}
$$

thus saying that the interparticle spacing is much higher than the range of the potential, to make very unlikely to have three particles in the interaction range. So we are only considering two-body elastic collisions ${ }^{6}$
Thirdly, the degeneracy condition is giving us a relation between the de Broglie wavelength and the density, $n \lambda_{d B}^{3} \approx 1$, and thus relating $\lambda_{d B}$ to the interaction range

$$
\begin{equation*}
\lambda_{d B} \gg r_{v d W} \tag{1.30}
\end{equation*}
$$

This is a very important relation, which is telling us that the scale of the interaction is much lower than the typical extent of the atoms, which is related to the typical momentum of collisions $k=2 \pi / \lambda_{d B}$. For this reason, when two atoms are colliding, they are not feeling all the details of the short-ranged potential, rather than an effective potential averaged over the extent of the whole wavefunction.
The main reference for this part is [17]. The Schrödinger equation in the reference frame of two colliding atoms can be generally written as

$$
\begin{equation*}
H|\psi\rangle=\left[H_{0}+V(\mathbf{r})\right]|\psi\rangle=E|\psi\rangle \tag{1.31}
\end{equation*}
$$

[^5]where $H_{0}$ is the free particle Hamiltonian ${ }^{[7}$ and $V(\mathbf{r})$ the interaction potential. For a short-ranged interaction, we can express the wavefunction away from the target region 8
\[

$$
\begin{equation*}
\psi_{\mathbf{k}}(\mathbf{r})=e^{i \mathbf{k} \cdot \mathbf{r}}+f(k, \theta, \varphi) \frac{e^{i k r}}{r} \tag{1.32}
\end{equation*}
$$

\]

i.e. as the sum of an incident plane wave in direction $\mathbf{k}$ and a scattered spherical wave: $f(k, \theta, \varphi)$ is the scattering amplitude modulating the amplitude and phase of the scattered components in the direction given by angles $(\theta, \varphi)$. The scattering amplitude is related to the scattering cross section $\sigma$ through the differential relation $\frac{d \sigma}{d \Omega}=\frac{4 \pi}{k^{2}}|f(\theta, \varphi)|^{2}$, where $\Omega$ is the solid angle.
Thanks to central symmetry of the scattering potential, the Hamiltonian $H$ is commutes with the angular momentum $L$, and thus admits a factorization of the radial part of the wavefunction from its angular one. The scattered wavefunction is in this way axially symmetric with respect to the incident wavevector $k$, and is thus written as

$$
\begin{equation*}
\psi_{\mathbf{k}}(\mathbf{r})=\sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} Y_{l, m}(k, \theta, \varphi) \frac{\left.u_{k, l, m}(r)\right)}{r} \tag{1.33}
\end{equation*}
$$

where $Y_{l, m}(k, \theta, \varphi)$ are the eigenfunctions of the angular momentum $L$. The Eq. 1.31) is thus equivalent to

$$
\begin{equation*}
\left.\left.u_{k, l, m}^{\prime \prime}(r)\right)+\left[k^{2}-\frac{l(l-1)}{r^{2}}-\frac{2 m}{\hbar^{2}} V(r)\right] u_{k, l, m}(r)\right)=0 \tag{1.34}
\end{equation*}
$$

which has to be satisfied for every $k, l, m$. The splitting of the Schrödinger equation in infinite decoupled equations for each value of the angular momentum is the mathematical way to express the physical features of conservation of angular momentum in central fields. If we assume $z$ axis to be parallel to $\mathbf{k}$, because the incident plane wave has $m=q^{9}$, through partial wave expansion [18] of the scattering amplitude, it can be showed that all terms with $l \neq 0$ are negligible.
There is a simple heuristic explanation of the only $s$-wave $(l=0)$ scattering contribution: the effective potential in Eq. 1.34 is made of the two terms

$$
\begin{equation*}
V_{e f f}(r)=V(r)+\frac{\hbar^{2}}{2 m} \frac{l(l+1)}{r^{2}} \tag{1.35}
\end{equation*}
$$

i.e. the short-ranged potential term and the centrifugal barrier. For small $k$ values e.g. as in ultracold gases, the particles with $l>0$ cannot penetrate the centrifugal barrier below the potential range $r_{v d W}$ (see Fig. 1.5) and thus they aren't experiencing the scattering potential $V(r)$. On the other hand, the s-wave term don't feel any centrifugal repulsion,

[^6]

Figure 1.5: Effect of the centrifugal barrier for $l>0$ (in blue) on the Lennard-Jones-like potential of the atoms (in green), which is the one for $l=0$. The barrier prevents the atoms to approach at short distances, quenching collisions.
thus the isotropic s-wave channel is the only scattered in small $k$ limit. The scattering amplitude reduces to [18]

$$
\begin{equation*}
f \approx f_{s-w a v e}=\frac{1}{k \cot \left[\delta_{0}(k)\right]+i k} \approx \frac{1}{a^{-1}+r_{e f f} \frac{k^{2}}{2}+i k} \tag{1.36}
\end{equation*}
$$

where, in the last step, we have expanded the term $k \cot \left[\delta_{0}(k)\right]$ to the second order. The collision imprints a phase shift $\delta_{0}(k)$ to the s-wave channel of the scattered wavefunction, which can be parametrized only by the effective range $r_{\text {eff }}$ of the potential and the scattering length $a$, defined by

$$
\begin{equation*}
a=\lim _{k \rightarrow 0} \frac{\tan \left[\delta_{0}(k)\right]}{k} \tag{1.37}
\end{equation*}
$$

Despite both $a$ and $r_{e f f}$ are set by microscopic details of the potential, the same values of scattering length and effective range of potential can be obtained starting from very different microscopic interaction potentials; this allows to replace the not exactly known microscopic potential with a much simpler effective one, the so-called pseudo-potential, with the same asymptotic behaviour at long distances [19].
Taking in account the indistinguishability among particles, we must (anti-)symmetrize the wavefunction for (fermions) bosons, which leads to the correct quantum differential cross section expression for indistinguishable particle:

$$
\begin{equation*}
\left\{\frac{d \sigma}{d \Omega}\right\}_{i d}=|f(\theta) \pm f(\pi-\theta)|^{2}, \quad 0<\theta<\pi / 2 \tag{1.38}
\end{equation*}
$$

In the end, we can write the cross section in the limit $k \rightarrow 0$ (18):

$$
\left\{\begin{align*}
\sigma_{i d, b o s} & =\frac{8 \pi a^{2}}{1+k^{2} a^{2}}  \tag{1.39}\\
\sigma_{i d, f e r} & =0 \\
\sigma_{\text {non id }} & =\frac{4 \pi a^{2}}{1+k^{2} a^{2}}
\end{align*}\right. \text { (Pauli principle) }
$$

The second equation is a consequence of the Pauli exclusion principle: indeed, two identical fermions have the same spin value, thus a symmetric spin-wavefunction; but the $s$-wave channel of the scattering implies they to have also a symmetric wavefunction, thus they cannot interact. The lower energy interaction allowed for two identical fermions is the $l=1$ ( p -wave configuration), which have a centrifugal barrier with maximal height of $600 \mu \mathrm{~K}$ for the ${ }^{6} \mathrm{Li}$. This is a very important features of fermions: while on one hand this requires at least two species to perform evaporative cooling, on the other hand in this way three body losses of Fermi gases are suppressed because they can only be realized by the unlikely $l>0$ scattering configurations.
From Eq. 1.39), we can get at least two interesting limits:

- $k|a| \ll 1$ and $k r_{e f f} \lesssim 1$ : in this conditions, the entire physics of the system depends on one only parameter, $a$, because $f$ depends only on the scattering length: $f=-a$. Since under the effect of any short range potential on the wavefunction can be modeled at low energies as the effect of a hard-core potential with core radius equal to scattering length [20], the interaction can be described using the contact pseudo-potential

$$
\begin{equation*}
V(r)=g \delta^{3}(r) \tag{1.40}
\end{equation*}
$$

where the coupling constant is defined as

$$
\begin{equation*}
g=\frac{4 \pi \hbar^{2} a}{m} \tag{1.41}
\end{equation*}
$$

Eq. (1.39) thus becomes

$$
\begin{cases}\sigma_{i d, b o s} & =8 \pi a^{2}  \tag{1.42}\\ \sigma_{i d, f e r} & =0 \\ \sigma_{\text {non id }} & =4 \pi a^{2}\end{cases}
$$

Eq. (1.42) cannot hold in general, because the case of divergent scattering length leads to an unphysical divergence of the total scattering cross section.

- $k|a| \gg 1$ and $k r_{\text {eff }} \ll 1$ : this, which is called unitary limit, is the case with the highest interactions. The scattering amplitude becomes from Eq. 1.36 $f=i / k$; the resultant total scattering cross section for fermions becomes from Eq. (1.39)

$$
\begin{equation*}
\sigma_{\max }=\frac{4 \pi}{k^{2}} \tag{1.43}
\end{equation*}
$$

which is the maximum for the s-wave collisions. The divergence of the scattering length is only possible when a bound state is supported by the interaction potential.


Figure 1.6: On the left, the two different open (red) and closed (blue) channels. The lower grey dashed line is the energy of the open channel of the scattering event, while the upper is the energy of the bound state. On the right, the energy difference between the two lines is written as function of the external magnetic field: for some value of the field $B_{0}$, the two states are becoming energetically degenerate.

This scenario, which seems to be a textbook exercise, can be actually implemented in a quantum gas thanks to the magnetic Feshbach resonances [21], allowing us to "tune" the scattering length of the atoms.

## Feshbach resonances

The interaction between two scattering atoms is mediated by some kind of potential which depends on their internal states e.g. for ${ }^{6} \mathrm{Li}$, the electrons of two different atoms in the ground state ${ }^{2} S_{1 / 2}, F=1 / 2 m_{F}= \pm 1 / 2$ can form, during a scattering event, a singlet or triplet state. The conservation of energy in the elastic collisions allows scattering processes only in the so-called open channels, i.e. for inner states interactions which are energetically accessible. The energetically forbidden channels are on the other hand called closed channel.
If the open and closed channels states have different magnetic momenta e.g. as in a singlet and triplet state, the energy difference between them can be tuned by adjusting an external magnetic field: if the closed channel potential admits a bound state, thanks to magnetic field it can become degenerate i.e. resonant with the energy of the open channel (see Fig. 1.6). In this way, the two states are mixed, and the bound state affects the continuum scattering channel with a resulting effect of tuning the scattering length: this phenomenon is called Feshbach resonance.
Usually, the scattering lenght $a$ is depending on the magnetic field $B$ according to equa-
tion

$$
\begin{equation*}
a(B)=a_{b g}\left(1+\frac{\Delta B}{B-B_{0}}\right) \tag{1.44}
\end{equation*}
$$

where $a_{b g}$ is the background i.e. without magnetic field scattering length, $\Delta B$ and $B_{0}$ are the width and the field respectively of the resonance (see $a(B)$ plot in Fig. 1.8.
The broadness of the Feshbach resonance affects the effective range of the potential. For alkali atoms, $r_{e f f}$ can be written as

$$
\begin{equation*}
r_{e f f}=\frac{\hbar^{2}}{2 \mu a_{b g} \Delta B \delta \mu} \tag{1.45}
\end{equation*}
$$

where, $\mu$ is the reduced mass and $\delta \mu$ the difference between the magnetic momenta of open and closed channels. If this quantity is smaller than the var der Waals range, the scattering problem is decoupled for the potential details and with a universal character. In Eq. 1.36, we can thus neglect the effective range term; in this case, the scattering amplitude has a pole at an energy

$$
\begin{equation*}
E_{b}=\frac{\hbar^{2}}{2 \mu a^{2}} \tag{1.46}
\end{equation*}
$$

which is the binding energy $E_{b}$ of the Feshbach molecules. More details about Feshbach resonances can be found in
citechin:feshbach.

## Lithium resonances

The ${ }^{6} \mathrm{Li}$ Feshbach resonances are among the broadest ones observed in cold atoms for all the three lowest hyperfine states (in Fig. 1.7 are showed the ground state hyperfine states behaviour in a magnetic field): as a consequence, we have a very small effective range of the potential, and a precisely tunable scattering length.
The three lowest energy states in Paschen-Bach limit are usually labeled as $|1\rangle,|2\rangle$ and $|3\rangle$ and are corresponding to the states:

- $|1\rangle=\left|F=1 / 2 ; m_{F}=-1 / 2\right\rangle$ at low field and $|1\rangle=\left|m_{I}=+1 ; m_{J}=-1 / 2\right\rangle$ at high field;
- $|2\rangle=\left|F=1 / 2 ; m_{F}=+1 / 2\right\rangle$ at low field and $|1\rangle=\left|m_{I}=0 ; m_{J}=-1 / 2\right\rangle$ at high field;
- $|3\rangle=\left|F=3 / 2 ; m_{F}=-3 / 2\right\rangle$ at low field and $|1\rangle=\left|m_{I}=-1 ; m_{J}=-1 / 2\right\rangle$ at high field;

The Feshbach resonance we have used during this thesis is the one between state $|1\rangle$ and $|2\rangle$, which has these parameters [22]:

| $B_{0}[\mathrm{G}]$ | $\Delta B[\mathrm{G}]$ | $a_{b g}\left[a_{0}\right]$ |
| :---: | :---: | :---: |
| 832 | 262 | -1582 |



Figure 1.7: Splitting on ${ }^{6}$ Li hyperfine levels due to angular momentum: at no or low magnetic field (Zeeman regime), the atom experiments hyperfine splitting, while at high fields (Paschen-Bach regime) is dominating the coupling between field and total electronic angular momentum $J$. The high fields regime in ${ }^{6} \mathrm{Li}$ is already reached around 100 G . Figure taken from [12
where $a_{0}$ is the Bohr radius. In Fig. 1.8 , the plot of $a(B)$ is showed, and the consequent different on the many-body behaviour of a degenerate gas thanks to the different scattering length.
One of the most interesting features about tuning the scattering length and thus interactions through Feshbach resonances indeed is the effect on the many-body behaviour: this leads to the BEC-BCS crossover. Some references in which this phenomenon is theoretically described and experimentally investigated are [23, 16]. Here, I treat briefly some aspects connected with the experiments performed in this work.
Tunable interactions leads to the possibility to study the behaviours of the many-body state between the two paradigmatic regimes of superfluidity: a fermionic BCS superfluid of long-range Cooper pairs on the attractive side of the Feshbach resonance ( $a \lesssim 0$ ) and a Bose-Einstein condensate of tightly bound molecules (mBEC) on the repulsive branch ( $a \gtrsim 0$ ). In the intermediate regime, for diverging scattering length on top of the resonance, we have a Unitary Fermi Gas (UFG); with the scattering length dropping out of the problem as a scale, the only relevant scales are set by the Fermi energy $E_{F}$ and the Fermi momentum $k_{F}$, and thus the gas behaviour is universal. Another important


Figure 1.8: The scattering length close to the Feshbach resonance as function of the magnetic field. Sketched there are the three regimes of superfluid flow for the values of the scattering length: BEC in the $a>0$ region, UFG when $a->\infty$, BCS in the $a<0$ region.
feature, as seen in Eq. 1.43, is having the maximum of scattering cross section: the unitary gas, indeed, is the best condition to perform evaporative cooling because of the fastest thermalization rate.
The mBEC was the system I used to perform the measurements of this thesis, as discussed in Chap. 3. On the repulsive side of the Feshbach resonance, the pairs of atoms are converted to deeply bound molecules, and for sufficiently large binding energy, when their size becomes much smaller than the inter-particle distance, these composite molecules behave as bosons. The scattering length of dimer-dimer scattering, in this limit, has be found 24 to be $a_{d}=0.6 a$, thus resulting in a stabilizing repulsive interaction with respect to collapse into more deeply bound molecules.
When the temperature is going down, these dimers are undergoing Bose-Einstein condensation [25]. For trapped gases with weakly repulsive interactions, the critical temperature for condensation can be expressed as 26

$$
\begin{equation*}
T_{c}=\frac{\hbar \bar{\omega}}{k_{B}}\left[\frac{N}{\zeta(3)}\right]^{1 / 3} \tag{1.47}
\end{equation*}
$$

where $\zeta(n)$ is the Riemann zeta function.
Below $T_{c}$, the system occupies macroscopically the lowest energy level, and the ratio between the number of atoms in the lowest level $N_{0}$ and the total atoms number $N$ can
be expressed by

$$
\begin{equation*}
\frac{N_{0}}{N}=1-\left(\frac{T}{T_{c}}\right)^{3} \tag{1.48}
\end{equation*}
$$

The density profile of the trapped gas can be obtained using the Gross-Pitaevskii equation for a weakly interacting gas [26]: in this case, using the contact pseudo-potential of Eq. 1.40 and in the Thomas-Fermi approximation 10 , the GPE are resolved by a wavefunction corresponding to a parabolic inverted profile:

$$
\begin{equation*}
n_{T F}(\mathbf{r})=\max \left[\frac{\mu}{g}\left(1-\frac{x^{2}}{R_{x, F}^{2}}-\frac{y^{2}}{R_{y, F}^{2}}-\frac{z^{2}}{R_{z, F}^{2}}, 0\right]\right) \tag{1.49}
\end{equation*}
$$

where $\mu$ is the chemical potential, $g$ is the coupling constant of the pseudo-potential defined by Eq. 1.41 and $R_{i, F}=\sqrt{2 \mu /\left(m \omega_{i}\right)}$ is the Thomas-Fermi radius in the $i$ direction. Calculating the chemical potential for a harmonic trap, it can be written as function of trap parameters $\bar{\omega}$, geometric mean of the trapping frequencies, and $a_{T}=$ $\sqrt{\hbar /(m \bar{\omega})}$, trap dimension [26]:

$$
\begin{equation*}
\mu=\frac{\hbar \bar{\omega}}{2}\left(\frac{15 N a}{a_{T}}\right)^{2 / 5} \tag{1.50}
\end{equation*}
$$

where $N$ is the total number of atoms and $a$ the scattering length.

### 1.3 Producing a degenerate Fermi gas

In this section, I do a brief overview of the whole apparatus and routine to cool down and trap the ${ }^{6} \mathrm{Li}$ gas. The main reference for this overview is the PhD thesis 14$]$.

### 1.3.1 Bringing atoms into science chamber

The first demanding issue in experiments of this kind is the isolation from the thermal background of atoms and molecules, which requires an Ultra-High Vacuum (UHV) apparatus, while on the other hand the ${ }^{6} \mathrm{Li}$ sample needs to be heated to a temperature over $400^{\circ} \mathrm{C}$ to extract a significant vapour pressure. A sketch of the UHV system is shown in Fig. 1.9 .
The ${ }^{6} \mathrm{Li}$ sample is heated in a diffusive oven, with cup maintained at a temperature of $420{ }^{\circ} \mathrm{C}$; the cup is connected to the main oven chamber by a circular nozzle, which also collimates the vapour travelling through the system. The nozzle is held at a temperature of $460{ }^{\circ} \mathrm{C}$, to avoid sticking and solidification of Lithium on it. Then, for further collimating the atomic beam, a copper cold finger is placed after the nozzle.
To guarantee a sufficient lifetime to the atomic cloud in the science chamber, a pressure below $10^{-11} \mathrm{mbar}$ is required. To avoid pressure contaminations from the oven, atoms

[^7]

Figure 1.9: 3D sketch of the whole UHV setup.
go through a differential pumping stage entering in the Zeeman slower.
The Zeeman slower allows for decelerating the atoms of the collimated beam by the means of a counter propagating laser beam and an inhomogeneous magnetic field. The combined effect of this two features is reducing velocity from about $800 \mathrm{~m} / \mathrm{s}$ to a value around $60 \mathrm{~m} / \mathrm{s}$, and increasing in this way the magneto-optical trap (MOT) capture efficiency. The magnetic field is needed to keep the atoms in resonance with the laser beam, compensating for the changes in Doppler shift and allowing a continuous cooling. Thus, the field shape can be obtained from the condition:

$$
\begin{equation*}
\omega_{L}-\omega_{0}+k_{L} v=\frac{\Delta E_{h s}(B)}{\hbar} \tag{1.51}
\end{equation*}
$$

where $\omega_{L}$ is the beam frequency, $\omega_{0}$ is the atomic resonance at zero field and $\Delta E_{h s}(B)$ is the hyperfine splitting of the cooling transition as function of the magnetic field. As the ${ }^{6}$ Li atoms enter in the Paschen-Back limit at relatively low fields, we can approximate $\Delta E_{h s}(B)=\mu_{B} B$, obtaining the expression for the magnetic field:

$$
\begin{equation*}
B(z)=\frac{\hbar}{\mu_{B}}\left(\Delta_{0}+k_{L} \sqrt{v_{i}^{2}-2 a z}\right) \tag{1.52}
\end{equation*}
$$

where $\Delta_{0}=\omega_{L}-\omega_{0}, v_{i}$ is the maximum velocity slowed down and $a$ is the deceleration, which can be evaluated by solving the optical Bloch equations.
The Zeeman slower in our experiment is in the so-called spin-flip configuration, since the magnetic field direction is inverted through the tube; the disadvantage of this configuration is the needing of a repumping ligth to recover atoms after depolarization due to the arbitrariness of the angular momentum axis at zero field. On the other hand, this configuration presents two main advantages: firstly, the atoms at the exit of the slower, are


Figure 1.10: 3D sketch of the science chamber, with both the Feshbach (in brown) and MOT (in purple) coils. For the sake of clarity, there's only the lower of the two MOT coils.
off-resonance with the counter-propagating beam; secondly, a lower power consumption is required.
At the other side of the Zeeman slower is placed the science chamber, in which all cooling stages, from the MOT to the evaporative cooling, and all experiments are performed. The chamber is an octagonal stainless-steel cell, with optical access from different directions thanks to several windows. In the vertical direction, it's equipped with two large re-entrant viewports, made of silica with 60 mm diameter and 6 mm thickness: the two vertical windows are relatively distant 25.4 mm . All this allows to have a large numerical aperture and to place a high resolution imaging objective close to the atoms, as was implemented in last months. In Fig. 1.10 is shown a sketch of the science chamber, with also the coils used for the MOT and the Feshbach field.

### 1.3.2 Reaching degeneracy

Once atoms are in the science chamber, we need to trap and cool down them. To this purpose, there are several steps: the first one is the magneto-optical trap (MOT), then there is a step of $\mathrm{D}_{1}$ gray molasses cooling to a better loading in the optical dipole trap (ODT) and the last stage is the evaporative cooling from the optical trap.
The laser sources needed in our experiment for all this stages are three: a red laser optimized for emission at wavelength $\lambda_{D 2}=670.977 \mathrm{~nm}$, which corresponds to $\mathrm{D}_{2}$ transition of ${ }^{6} \mathrm{Li}$ atoms; a red laser optimized for emission at wavelength $\lambda_{D 1}=670.979 \mathrm{~nm}$, which corresponds to $\mathrm{D}_{1}$ transition of ${ }^{6} \mathrm{Li}$ atoms; an IR laser at wavelength $\lambda=1064 \mathrm{~nm}$, red-detuned with respect to the most intense transitions for the ODT. The red lasers, beyond the MOT and the grey molasses, are used also as the off-resonance light for the

Zeeman slower.
For the Zeeman slower, the MOT and the grey molasses, we need a stable frequency of the red light, while air flows, electronic noise and temperature drifts can cause a drift in the frequency of the laser. To the purpose of stable laser frequency, we use Dopplerfree saturation absorption spectroscopy to lock the cavity of the lasers, thanks to beams passing through an heat pipe at $330^{\circ} \mathrm{C}$ with ${ }^{6} \mathrm{Li}$ enriched sample.

## Magneto-optical trap

Atoms going out from the slower are captured in a trap made with a magnetic quadrupole field and counter-propagating slightly red-detuned laser beams in the three axis directions, a magneto-optical trap (MOT). The three beams are red-detuned to be resonant with atoms moving towards them in all directions, thus the effective force filled by one two-levels atom from a couple of counter-propagating beams is a viscous force $F_{m o l}=-2 \alpha k v$, where $k$ is the wavevector of the light, $v$ is the velocity of the atom and $\alpha$ is a constant proportional to linewidth of the transition, intensity and detuning of the laser; this result can be understood as an effect of the radiative force introduced in Sect. 1.1.1. Accordingly, the six laser beams are slowing down the atoms, and cooling down the cloud; in our experiment, every couple of beams is realized with only one beam retroreflected from a mirror out of the chamber.
While lasers are required to cool down atoms, we need a magnetic quadrupole field to trap them: the splitting of levels with different angular momentum, due to the magnetic field, combined with the lasers, has a confining effect on the atoms, originating an effective linear force of the form $F_{\text {trap }}=-2 \alpha \beta x$ on every axis, where $\beta$ is proportional to the gradient of the magnetic field. To get this confinement, the two counter-propagating beams needs to have opposite circular polarization. The quadrupole field in our experiment is generated by a pair of coils in anti-Helmholtz configuration, whose axis is the z-axis of the chamber, as shown in Fig. 1.10 .
For the MOT, we're using the $\mathrm{D}_{2}$ laser, both because this transition requires lower laser power because of the lower saturation intensity compared to the $\mathrm{D}_{1}$ transition and because, theoretically, the only closed transition in the ${ }^{6} \mathrm{Li}$ system is the $F=3 / 2 \rightarrow 5 / 2$ transition, contained in the hyperfine structure of the $\mathrm{D}_{2}$ line. However, for having unresolved hyperfine structure, we need to trap and cool down atoms in the MOT a repumping light from the state with $F=1 / 2$, like for the others alkali atoms.
In our experiment, we typically collect some millions of atoms in the MOT, at a temperature around 1 mK . The temperature at this stage is limited by the absence of an efficient sub-Doppler cooling because of the unresolved hyperfine structure of the excited level.

## $\mathrm{D}_{1}$ gray molasses cooling

As described in details in [27], in our experiment an efficient sub-Doppler scheme based on gray molasses, which exploits the $\mathrm{D}_{1}$ transition, has been developed. The sub-Doppler name derives from the temperature lower the Doppler limit for the $\mathrm{D}_{2}$ transition of ${ }^{6} \mathrm{Li}$,
which is

$$
\begin{equation*}
T_{D}=\frac{\hbar}{2 k_{B}} \Gamma \approx 140 \mu \mathrm{~K} \tag{1.53}
\end{equation*}
$$

This stage is using the $D_{1}$ laser and the cooling mechanism relies on the coupling of dark and bright states in a Sysyphus-like cooling: the efficiency is driven by relative intensity ratio and relative detuning between cooling and repumping light. Exploiting this system, we can reach the temperature of about $50 \mu \mathrm{~K}$ with about the $75 \%$ of atoms loaded in the MOT. The $\mathrm{D}_{1}$ is also useful because at the end the atoms can be pumped in the state $F=1 / 2$ switching off some tens of $\mu$ s the rempumping light after the cooling one. This is increasing moderately the temperature, about $10 \%$.

## Evaporative cooling: high power ODT

The last stage is the evaporative cooling: atoms, cooled and optically pumped in the state ${ }^{2} \mathrm{~S}_{1 / 2}, F=1 / 2$, are loaded in an optical trap made by the use of high power infrared lasers, then evaporated lowering the trap thresholds and letting atoms thermalize.
After $\mathrm{D}_{1}$ cooling stage, we load the atoms in an ODT made of one single focus laser beam at high power (IPG laser), feedbacked with a PID controller. The waist diameter is about $40 \mu \mathrm{~m}$, while the power is usually around 130 W above a maximum of 200 W . At maximum power, the trap depth is very deep with respect to the temperature of the atomic cloud. For this reason, we apply a fast sinusoidal modulation on the Acousto-Optical Modulator (AOM) radiofrequency input, both frequency and amplitude: varying the radiofrequency injected in the $A O M$, we are also varying the direction of diffraction of the beam very fastly. For the modulation frequency being above the trapping frequencies, the atoms feel a time-averaged dipole potential with a larger effective waist, about $70 \mu \mathrm{~m}$, which corresponds to a trap depth of around 1 mK . The modulation frequency is above the PID bandwidth, so the feedback action isn't influenced by the presence of the modulation. The laser is switched on at high power after the $\mathrm{D}_{1}$ cooling, and in the trap up to 10 millions of atoms are loaded; then, there is a first evaporation exponential ramp ( 250 ms duration time), which leads the laser to a power around 30 W . At this point, there is another infrared laser switched on (Mephisto laser), crossed with the IPG at an angle of $14^{\circ}$, with a circular waist of about $45 \mu \mathrm{~m}$. From this moment on, the trap becomes a crossed-beams dipole trap.
The degeneracy is reached during the second evaporation ramp of both IR lasers, which lasts 4 s or 6 s : after a first part with exponential reducing of IPG and constant Mephisto, the second part is made of an exponential for both lasers, with a constant ratio between the two laser powers. In some cases, at the end of the evaporation there is a recompression step, to avoid a too low trap depth.
The evaporation is performed with the two states of the $F=1 / 2$ manifold, $|1\rangle$ and $|2\rangle$, as defined in Sect. 1.2 .3 . The magnetic field from the Feshbach coils is splitting the atomic population onto the two states: this is a key point of the evaporation of the fermions, which is efficient only if atoms are populating different states. Unless, as discussed in Sect. 1.2.3, the Pauli principle will suppress the collisions in the gas, avoiding the thermalization of the cloud and so on the cooling. On the other hand, to enhance


Figure 1.11: a. Laser scheme in the xy plane (brown beams are 1064 nm beams for ODT). b. Laser scheme on the vertical axis.
collisions, we evaporate the atoms at Feshbach resonance field for the pair $|1\rangle-|2\rangle$; at the resonance, the scattering cross-section is maximized, while the Pauli exclusion principle forestalls three-body losses, increasing the lifetime of the sample. Close to the end of the evaporation, we perform a sweep of the magnetic field to a target field on the repulsive branch of the Feshbach resonance, obtaining a BEC superfluid of ${ }^{6} \mathrm{Li}$ molecules (mBEC). In Fig. 1.11, there is a scheme of all beams going through the science chamber, both in xy plane and on the z axis. Once we have obtained a degenerate gas, then we can apply a dipole potential to study different kinds of physics: next step is the design and implementation on the experiment of different potentials.

### 1.3.3 Imaging the atomic cloud

The imaging is the way to acquire information about our gas sample contained in the science chamber: in ultra-cold atoms, the most important observable in the system is the density $n(x, y, z)$. The technique exploited in our apparatus is the absorption imaging i.e. imaging the shadow of the atomic cloud illuminated by resonant light.

The fluorescent power i.e. the rate of power emitted by $N$ atoms considered as two-level systems coupled with the imaging light at frequency $\omega$ in the steady state can be written as 28]:

$$
\begin{equation*}
P_{F}=N \hbar \omega_{0} \frac{\Gamma}{2} \frac{s}{1+s} \frac{1}{1+\left(\frac{\delta}{\sqrt{1+s}}\right)^{2}} \tag{1.54}
\end{equation*}
$$

where $\omega_{0}$ is the frequency of the atomic resonance, $\Gamma$ is the linewidth of the atomic transition, $\delta=\frac{\omega-\omega_{0}}{\Gamma / 2}$ is the detuning expressed in half linewidth units and $s=I / I_{\text {sat }}$ is the saturation parameter, defined as the ratio between the imaging field intensity and the transition saturation intensity

$$
\begin{equation*}
I_{s a t}=\frac{\hbar \Gamma \omega_{0}^{3}}{12 \pi c^{2}} \tag{1.55}
\end{equation*}
$$

From Eq. 1.54 and conservation of energy, one can compute the drop in the laser beam intensity crossing the atomic cloud. After travelling a distance $d z$ in the gas, the decrease in light intensity $d I$ is

$$
\begin{equation*}
d I=-\frac{1}{A} P_{F}(d z)=-\hbar \omega_{0} \frac{\Gamma}{2} \frac{s}{1+s} \frac{1}{1+\left(\frac{\delta}{\sqrt{1+s}}\right)^{2}} n d z \tag{1.56}
\end{equation*}
$$

where $A$ is the laser beam cross section and $n$ is the atomic density. Solution to Eq. (1.56) links the intensity drop to the atomic density, which is exactly the purpose of the imaging. The imaging can be performed in the two regimes of low and high imaging light intensity, depending on the value of the parameter $s$ :

- for resonant frequency $\delta=0$ and low intensity $s \ll 1$, Eq. 1.56 can be approximated as

$$
\begin{equation*}
d I=-\hbar \omega_{0} \frac{\Gamma}{2} \frac{I}{I_{s a t}} n d z=-I \frac{3 \lambda_{0}^{2}}{2 \pi} n d z \tag{1.57}
\end{equation*}
$$

Integrating this equation along the $z$ axis, which is the axis of propagation of the imaging beam, one obtains the relation

$$
\begin{equation*}
I(x, y)=I_{0}(x, y) e^{\sigma_{0} n(x, y)} \tag{1.58}
\end{equation*}
$$

where $I_{0}(x, y)$ is the imaging intensity profile before crossing the cloud, $\sigma_{0}=$ $\hbar \omega_{0} \Gamma /\left(2 I_{\text {sat }}\right.$ is the absorption cross section and $n(x, y)$ is the two-dimensional density of the atomic cloud, integrated along the $z$ direction

$$
\begin{equation*}
n(x, y)=\int_{-} \infty^{\infty} n_{3 D}(x, y, z) d z \tag{1.59}
\end{equation*}
$$

being $n_{3 D}(x, y, z)$ the density of the cloud in the real space. The integrated density is the only measurable quantity with absorption imaging, and from Eq. (1.58) is given by

$$
\begin{equation*}
n(x, y)=\frac{1}{\sigma_{0}} \log \left(\frac{I(x, y)}{I_{0}(x, y)}\right) \tag{1.60}
\end{equation*}
$$



Figure 1.12: Above, the absorption image of a BEC cloud. Below, the density distribution (light blue circles) with a bimodal fit (red solid line), with the characteristic edges due to the inverted parabola function.

The intensity profiles $I(x, y)$ and $I_{0}(x, y)$ can be measured using a CCD camera with two different imaging pulses, one with atomic cloud $I^{C}$ and one without it $I_{0}^{C}$. With the CCD, one needs to acquire also a background image without imaging light, to remove the offset introduced in the images due to dark current.

- for resonant frequency $\delta=0$ and high intensity $s \gtrsim 1$, the integrated density profile is obtained integrating directly Eq. (1.56) as (see Ref. [29] for details)

$$
\begin{equation*}
n(x, y)=\frac{1}{\sigma_{0}}\left[\log \left(\frac{I^{C}(x, y)-I_{b g}^{C}(x, y)}{I_{0}^{C}(x, y)-I_{b g}^{C}(x, y)}\right)-\frac{I^{C}(x, y)-I_{0}^{C}(x, y)}{I_{s a t}^{C}}\right] \tag{1.61}
\end{equation*}
$$

where $I_{\text {sat }}^{C}$ is a CCD parameter calibrated for an imaging beam intensity $I_{0}=I_{\text {sat }}$.
Fig. 1.12 shows a highly magnified absorption image of the density profile of a ${ }^{6} \mathrm{Li}$ molecules BEC cloud. The characteristic bimodal (Gaussian for the thermal component of the gas and inverted parabola from Eq. (1.49) for the BEC) density distribution is clearly visible.

## Chapter 2

## Tailored optical potentials on atoms

In this chapter, I introduce the tools of our experimental apparatus to produce a highresolution tailored optical potential: a high-resolution objective, used both for vertical imaging and for imprinting optical potentials onto the atomic samples, and a Digital Micromirror-Device (DMD).
The scales we are interested in resolving with our imaging system and manipulating with our tailored optical potentials are the ones of correlation length of atoms in a degenerate ${ }^{6} \mathrm{Li}$ fermionic gas, $1 / k_{F} \lesssim 1 \mu \mathrm{~m}$. While the gas is imaged by a resonant red laser beam, the potential are produced by the means of a green beam: as a consequence, the objective and its imaging system were designed to guarantee a resolution $R \lesssim 1 \mu \mathrm{~m}$ both for green and red light.

### 2.1 The high-resolution objective

We installed in our experimental setup a high-resolution custom objective, manufactured by Special Optics (3D sketch and all specifications in Fig. 2.1). We took advantage


Figure 2.1: 3D sketch of the objective. On the right, the objective specifications required to the manufacturer. The working distance is including the 6 mm silica viewport.


Figure 2.2: Image acquired by the CCD (left figure) and fit (right figure) of the two different imaged spots obtained due to lateral color aberration.
of the re-entering windows of our cell in the vertical direction to put the objective with high numerical aperture $(\mathrm{NA}=0.45)$ very close to the atom cloud: the working distance is 25.1 mm , with a field of view of 0.3 mm . The objective is coated for all wavelength we are using in our experiment ( $532 \mathrm{~nm}, 671 \mathrm{~nm}, 1064 \mathrm{~nm}$ ).
Before installing the objective, we measured its resolution both for red and green light: to this purpose, we illuminated an optical target made by a perforated metal plate, with a declared diameter of the holes of about 200 nm , well below of the diffraction limit of the objective (maximum resolution at the diffraction limit is $R_{\max }=\frac{0.61}{N A} * \lambda \approx 1.4 \lambda$, which corresponds to about 920 and 740 nm for the 671 and 532 nm wavelength light respectively). The optical path was made of a fibre with the collimator, the target and the objective mounted on a motorized translational stage (T-stage), then an iris, the tube lens ( $f=1000 \mathrm{~mm}$ ) we are actually using for the imaging, which is described later, and a CCD. The image acquired by the CCD corresponded to a distribution of the Point Spread Functions (PSFs) of the metal plate holes: fitting them with a Gaussian fit, we extracted the mean diameter of the PSF and thus the resolution of the imaging system. In particular, the key point is having resolution close to diffraction limit both for green and red light in one fixed distance between target and objective. Thus, we illuminated the target with both the wavelengths: due to lateral color chromatic aberration, the image of the same off-axis hole has two different position, depending on the wavelength (see Fig. 2.2 on the left). In Fig. 2.2 on the right, I show the Gaussian fit of the two PSFs, while in Tab. 2.1 I present the measured resolutions, which is compatible within

| Wavelength | Expected Resolution | Measured Resolution |
| :---: | :---: | :---: |
| 671 nm | 924 nm | $921 \pm 16 \mathrm{~nm}$ |
| 532 nm | 740 nm | $773 \pm 16 \mathrm{~nm}$ |

Table 2.1: Comparison of expected and measured resolution for green and red light. Errors are standard error of the fit.
two sigma with the expected one: the interesting result is that there is a configuration with diffraction-limited resolution of the imaging system for both red and green beam, such that we can have high-resolution imaging of the cloud while illuminating atoms with high resolution arbitrary potentials.
To be positioned, the objective was mounted on a 3 -axis translation stage (T-stage): the minimum step for this stage is of the order of some tens of nm. The T-stage itself was mounted on a tilt-pitch platform, to control the angle between the vertical beams and the objective. A 3D sketch of the objective placed into the experimental apparatus is shown in Fig. 2.3 the mirror used for the retroreflected MOT is inserted during the MOT by a motorized T-stage in a hole of the objective housing, designed at this purpose. After the MOT phase, the T-stage removes the mirror to free the path for imaging and green potentials. The MOT-mirror orientation can also be controlled thanks to a $x-y$ tilt mount equipped with micrometric actuators.
In Fig. 2.4 is showed the optical path from the objective to the atoms. Rigidly fixed to the objective, to reduce at minimum reciprocal shifts due to mechanical fluctuations of the setup, there are an iris (to increase the depth of field) and the tube lens. Closing the iris, we are able to reduce the NA of the imaging system, such to increase the depth of


Figure 2.3: 3D sketch of the objective placed inside the experimental apparatus.
field. The tube lens is an achromatic doublet, coated for both 670 nm and 532 nm light and with focal length of 1000 mm , letting us to have a nominal magnification $M=21.3$. After the lens, rigidly connected to it and to the objective too, there is a $45^{\circ}$ mirror, which reflects the beam horizontally. Another mirror follows the last one and sends the imaging beam towards an Andor Ixon3 EMCCD camera. In front of the camera there is an iris mounted on a black tube, to avoid stray light reflections to reach the CCD, and, at the end of the tube, a pair of blades, to reduce the portion of the illuminated chip and to allow us to perform fast kinetic acquisition.
The first necessary task to use the objective was a fine alignment of it with the respect to the chamber. To succeed in this task, firstly we needed an independent reference: thus, we aligned the vertical imaging beam, both to be orthogonal to the chamber windows and to hit the atomic cloud. To check the orthogonality of the beam, we used the retroreflection of the windows, and aligned it on the ingoing beam. To align the beam on the atomic cloud, we illuminated the atoms with the vertical beam while watching them in the horizontal direction: the vertical beam is centred on the cloud when the number of atoms seen in the horizontal imaging is the lowest. Then, we centred the T-stage, with the tube lens mount and the first mirror, on the imaging beam; mounting a mirror in the tube lens mount, we also checked a first time the tilting angle using the retroreflected beam; then, we checked also the collimation of the imaging beam.
After this preparation steps, we placed both the objective and the tube lens in their mounts. Then we aligned the objective position with a tiny target on the light before it, to be centred on the imaging beam; then, using two targets put on the tube lens and in the position of the first mirror, we aligned the tilt: the objective is aligned when the beam is hitting its centre and the power going through the two targets is maximized. After this, we put in the mirror and aligned it to have the beam reflected as horizontal as possible, to avoid astigmatism in the image. Last steps were putting the second mirror


Figure 2.4: Scheme of the optical path for the vertical imaging. The black dashed line indicates the change in projection plane.


Figure 2.5: Data (blue circles) and fit (solid line) of the position of the cloud on the Andor image as function of IPG last lens displacement. The slope extracted from the fit was used to compute the magnification.
and focusing the objective moving on the z -axis with the picomotor, watching at the dimensions of the atomic cloud image. To set better the tilt angle of the objective, we also left atoms to fall down without trap: letting them go out of focus, the objective was aligned to have the diffraction wings in the image symmetric. We needed to do several iterations of this centring, tilting and focusing routine, to reach the best working condition.
Once the objective is on focus, we calibrated the magnification of the new imaging system: we moved the final focusing lens of the IPG laser with a T-stage along the beam axis, moving in this way the position of the cloud. We calibrated the movement of the movement with the horizontal imaging, then we measured the displacement of the atoms in terms of sensor pixels on the image. Fitting the data as shown in Fig. 2.5, we obtained a magnification $M=21.8 \pm 0.3$. Thus, a pixel of the camera, which is $13 \mu \mathrm{~m}$ large, corresponds to about $0.6 \mu \mathrm{~m}$ in real space.

### 2.2 The Digital Micromirror Device

The other important tool implemented in the experimental apparatus was a Digital Micromirror Device (DMD), which allows fast and tailored creation of arbitrary potential. The device were previously characterized in a part of a master thesis performed in our lab [30], which is the main reference of this section. The DMD is a Spatial Light Modulator (SLM), a device that gives control over amplitude and phase of an incoming laser beam, and provides both truly static highly defined images and fast switching rates,


Figure 2.6: On the left, the two states of mirrors. On the right, picture of the DMD mounted on its support; a smile image is loaded on the board. Figure taken from 30
in case of a dynamical potential request.
The DMD mounted on the experiment is a DLP7000 Discovery ${ }^{T M} 4100$ 0.7" XGA 2xLVDS, produced by Vialux, with a V-7000 board. The device is composed by a $1024 \times 768$ array of mirrors, each one $13.68 \mu \mathrm{~m}$ large and tiltable on his diagonal axis of an angle $\pm 12^{\circ}$ with respect to the DMD surface, as shown in Fig. 2.6. The state of a single mirror is thus binary, and can be accessed using a computer connected with the board, to arrange the array in any kind of binary pattern to reproduce a black and white image. On the right of Fig. 2.6 is reported a picture of the DMD with an image loaded from the computer: the image is visible because of the different reflecting state of the mirrors.
As shown in Fig. 2.6, the DMD is rotated of $45^{\circ}$, to have the mirrors tilting axis to be


Figure 2.7: On the left, scheme of diffraction orders of the DMD. On the right, diffraction pattern from the DMD when the blazing condition is achieved. The rotation of the pattern of $45^{\circ}$ is due to rotation of the DMD itself. Figure taken from $\sqrt{30}$
vertical: in this way the main diffraction pattern is in the horizontal plane. Because of the micrometer-sized mirrors, comparable with the lasr wavelength, indeed the DMD behaves on the incoming laser beam as a 2D diffraction grating; actually, this is inconvenient, distributing the laser power over many diffraction orders and reducing the power hitting the atoms. Thus, in the thesis was found the blazing condition, for which every single mirror reflects the light in only one diffraction order, maximizing in this way the light power. This condition is showed in Fig. 2.7, where the most of power is in the central diffraction order: in the blazing condition, the usable power outgoing from the DMD is about $61.5 \%$ of the incoming one.

### 2.2.1 Optical path from DMD to the atoms

The DMD has been implemented within the lower breadboard, where also the Andor CCD camera for the vertical imaging is placed. The red beam for the imaging is combined with the DMD beam using a polarized beam splitter (PBS), as showed in Fig. 2.8 (where I sketched only one elliptic mirror after the tube lens for the sake of clarity); coming from the vacuum chamber, we also put a waveplate before the PBS to avoid intensity losses of the imaging beam on the other branch. To mix red and green light, we choose a PBS instead of a $45^{\circ}$ dichroic mirror because the mirror was reducing the resolution of the imaging system; the addition of PBS in principle is changing the magnification of the imaging system, but the measured variation was lower than $1 \%$, so under the experimental error.
The fiber is injected with a green laser Hermite-Gauss TEM $_{00}$ mode generated by a Verdi V-8 laser, made by Coherent; the injected beam has power around 1 W . The beam comes out from a Shäfter-Kirchhoff collimator, with a beam waist of about 5 mm . After the collimator, there is a dichroic mirror (DM) with high reflectivity for the green; we


Figure 2.8: Optical scheme of the path for the vertical imaging and for the DMD.
chose it to increase a bit the leak, which hits a photodiode (PD) and is used to stabilize the power with a PID controller. The DM reflectivity, on the other hand, depends on the polarization of the beam, thus a polarization fluctuations is converted into a power fluctuation on the PD: but a power decrease on the photodiode is a power increase on the whole DMD path, while the PID controller reads a lower intensity, enhancing the fluctuation. To avoid this unwanted effect, we put a PBS in transmission between the fiber collimator and the DM: indeed, the beam transmitted through a PBS has very clean polarization. The PBS converts any polarization fluctuation into a power fluctuation before the PID photodiode. Before the cube, a quarter-wave plate is placed, to convert any elliptical polarization to linear.
Then, the beam is going through a half-wave plate, to maximize the power reflected by the next PBS, which is going onto the DMD. The angle of the DMD is chosen to have the direction of the reflected beam to be collinear to the incoming beam: in this way, we avoided to cut the beam with the mounts of the PBS and of the DMD, but at the price to not fulfill perfectly the blazing condition, having the power in the most intense order to be a bit less than $61 \%$. Thus, passing twice through a quarter-wave plate, the polarization is rotated of $90^{\circ}$, so the beam is now transmitted by the cube. The cube is rigidly fixed with the next optics, to reduce fluctuations in relative position. After the PBS, the beam goes through a pair of lenses in $f+f^{\prime}$ configuration, with $f=250 \mathrm{~mm}$ and $f^{\prime}=100 \mathrm{~mm}$, so that they are realizing a demagnifying telescope for the image of about 2.5 times. The second lens is mounted together with a translator, thus we can set its position precisely to match the focus of the tube lens. After this lens, a flippable mirror is placed along the path: when the mirror is flipped out, the beam goes through a half-wave plate (to maximize the reflected power) and then to the 2 " cube, where it is reflected towards the atoms; when the mirror is inserted in the optical path, it's sending the beam to a CCD, used both for checking images and for calibration and images feedback purposes.
In the path there are two irises: the first one is used to select the only almost-blazed order of the DMD diffraction pattern, thus it's placed far enough from the DMD to have the orders well separated. The second iris is positioned in the focus of the first lens of the telescope, to act as a low-pass (LP) spatial filter: because of being in the Fourier plane of the first lens of a telescope, the iris is blocking part of the Fourier transform of the image focused by the lens. When the iris is well centred on the axis of the optical system, its aperture is setting the highest wavevector in the image, cutting the rest of the Fourier transform. Thus closing this iris, we can smooth the intensity profile of the image illuminating the atoms [30]. In Fig. 2.9 is shown the different shape of a barrier with the iris open and closed. As I'm going to explain later, this is also important to perform the images feedback.
Once the whole optics are placed, we needed to align the green beam to hit the atoms very precisely, because after the objective the dimension of the image on the atoms is the same order of the whole cloud dimensions: the image indeed is demagnified about 55 times. To have a precise alignment, we put two irises on the red beam, to reduce its diameter and to have a two different references, to align both angle and position. Then, we before used the red beam from the imaging, going through the cage, as a first



Figure 2.9: Difference in intensity shape of a barrier produced using the DMD between images acquired with opened or closed filtering iris. On the right, are showed the images; on the left, the intensity profile in the tight direction. Figure taken from 30
alignment reference. Then, with a retroreflecting mirror instead of the DMD, we aligned the green beam to be superimposed to the red one on the PBS, and we tilted the PBS to have the green beam going through the two irises on the vertical imaging beam.

### 2.2.2 Static optical potential with DMD

Once the DMD is placed with the whole optical path aligned, it was ready to be used to imprint an arbitrary potential on the atoms. Firstly, I briefly describe how to generate a static potential on the atoms, and the needed routine to calibrate the DMD and to feedback the images.
A static optical potential on the atoms consists in an image loaded on the DMD using the controlling software provided by Vialux, ALP basic GUI and illuminated by thw green beam. On the other hand, the intensity profile on the atomic plane isn't the same generated by the mirrors positioning, because the DMD acts as a light mask on the beam. Thus the image depends on the features of the Gaussian profile of the beam itself. Moreover, there also could be defects into the optical path, modifying the intensity profile. To avoid this problems, in the cited work [30] a feedback routine for the images was developed, based on a previous thesis performed in LENS Ytterbium laboratory [31]. The software was written down using a open source Python module by Sébastien Popoff [32], and it's working with two different steps: the first one is the required calibration between the image loaded on the DMD and the one acquired by the CCD; the second step is the actual feedback program. To calibrate the CCD means to find an affine transformationt to map the image acquired by the CCD on the image loaded on the DMD. In

[^8]

Figure 2.10: Difference between flat top profile images acquired before (upper left) and after (upper right) the application of the feedback routine. Lower on the left can be seen the change in the flat top intensity profile introduced by the feedback. Lower on the right the RMS as function of iteration of the feedback routine: filtering is reducing sensibly the RMS, thus it allows to have a more flat profile (see also Fig. 2.11. Figure taken from 30
our case, to this purpose we used a rotation and a translation. We loaded an image with three points in a well-known position on the DMD and compared with the position of the points in the CCD image: the getAffineTransform function was used to define the affine transformation matrix, which was applied to acquired images with the warpAffine function ${ }^{2}$. The matrix resulting from the application of the affine map to the acquired image is set to be $1024 \times 768$, to be compared with the DMD loaded one.
Once the CCD is calibrated, we ran the feedback program. The underlying principle of this Python routine is very simple: through the comparison between the target image and the acquired one, the error is estimated and used to correct the image loaded onto the DMD, then this operation is iterated. The error matrix of the $n$-th step $E_{n}$ is obtained through a pixel-by-pixel subtraction between the target image matrix $T$ and the $n$-th acquired one $C C D_{n}$, transformed by the calibration matrix $A$ :

$$
\begin{equation*}
E_{n}=T-A \cdot C C D_{n} \tag{2.1}
\end{equation*}
$$

[^9]

Figure 2.11: Effect of spatial filtering on a intensity profile generated by the DMD after the application of the feedback routine: only high spatial frequency modulations are affected by the filtering. Figure taken from 30

We are using two types of corrections, as in a PI controller: a proportional one, i.e. the addition of $E_{n}$ to the DMD matrix, weighted with a coefficient $k_{p}$, and an integral one, which keeps trace of all corrections performed during the feedback and it's done by adding all previous error matrices weighted with a coefficient $k_{i}$. The $(n+1)$-th image sent to the DMD is thus:

$$
\begin{equation*}
D M D_{n+1}=D M D_{n}+k_{p} E_{n}+k_{i} \sum_{m=0}^{n} E_{m} \tag{2.2}
\end{equation*}
$$

Furthermore, for each step, the RMS error is evaluated between the CCD acquired image and the target one:

$$
\begin{equation*}
R M S=\sqrt{\frac{1}{R \cdot C} \sum_{i, j}\left(\frac{C C D[i, j]-T[i-j]}{T[i, j]}\right)^{2}} \tag{2.3}
\end{equation*}
$$

where $R=768$ and $C=1024$ are the numbers of rows and columns respectively in the DMD image. Usually, we evaluate the RMS only in the non-zero region of the target image, to avoid saturation due to background noise. In Fig. 2.10 is shown the difference in the image and in the profiles before and after application of feedback routine. To flatten even more the image obtained with the feedback process, we can spatially filter the image illuminating the cloud by closing the iris 2 described in Sect. 2.2 .1 . As is shown in Fig. 2.11, this allow us to reduce the spatial high frequency modulations of the image, while affecting poorly the intensity profile ${ }^{3}$. The flatness of the potential, as discussed in Chap. 3, is strictly connected to heating effects of the imprinted potential.

[^10]
### 2.2.3 Dynamical control of DMD

The other very interesting feature of the DMD is the capability to display a sequence of images to create a time-dependent optical potential. In particular, we used this feature to apply for a fixed time a phase imprinting pattern (spatially constant o variable) on the cloud, as discussed in Chap. 3.
To use dynamically the DMD, we need to load the whole sequence of images in the DMD board RAM, with the SeqPut function, in the memory slot allocated by the SeqAlloc function. Using the DMD in Default mode, we have the trigger mode set to Master, in which the DMD is using an internal trigger for the images switch. In this case, with the SetTiming function we can choose the frequency of the images switch, with two main entries: the Illumination Time $I_{t}$, which sets the duration of one single image of the sequence, and the Picture Time $P_{t}$, which sets the interval between two consecutive dark phases (see Fig. 2.12 ). In this mode, before and after shining an image, the DMD has a dark phase, during which all the mirrors are flipped in the black position. This dark phase has to last at least $40 \mu \mathrm{~s}$ : for a sequence with $N$ images, if $P_{t}-I_{t} \geq 40 \mu \mathrm{~s}$, then the dark phase duration is set to shine every image for a time $I_{t}$ and to have the whole sequence to last $N \cdot P_{t}$; on the other hand, if $P_{t}-I_{t}<40 \mu \mathrm{~s}$, the Illumination Time set by the user is ignored and it's set such that the dark phase lasts $40 \mu \mathrm{~s}$, with whole sequence lasting $N \cdot P_{t}$ as before.
To our purpose, the dark phases of the Default mode are a problem, because in this phases the potential felt by the atoms is rapidly changing and can cause excitations in the cloud. To avoid this problem, we investigated a different operating mode, the Binary mode: the grayscale is no more available for the mirrors (i.e. the bitdepth is now 0 ), but the dark phases are kicked out of the images sequence. With DMD operating in this mode, the Illumination Time value is ignored and the duration of each image is set via the Picture Time; the minimum value that Picture Time could assume in this mode is $44 \mu$ s.
The operating Binary mode with Master trigger mode, is a good way when you don't need to choose the precise timing for each image to be on DMD. On the other hand, if you need a sharp control of the timing, the DMD is able to work with an external trigger, connecting to the board through a Molex connector. There are two operating modes with an external trigger: the Step and the Slave mode. In the Slave trigger mode, the DMD waits for a trigger to switch to next frame $f$ the sequence: the only variable we have to set is the Picture Time, the time after which the DMD search for a new trigger signal

## Dark phase <br> Illumination Time

$\xrightarrow{\square}$

Figure 2.12: Scheme of the Default dynamic operating mode, with Master trigger mode.


Figure 2.13: Above, scheme of the Slave trigger mode; in this case, the frame transition is supposed to be triggered by the rising of the signal. Below, scheme of the Step trigger mode; in this case, the frame transition is triggered by the high level of trigger signal.
(see Fig. 2.13 above). Thus, to have the DMD to work properly, the Picture Time has to be lower than the interval between two triggering signals; also in this case, $P_{t}$ cannot be lower than $44 \mu \mathrm{~s}$. With the DevControl, we can choose to be the rising or the falling of the external signal to trigger the image switch.
The Step mode, on the other hand, stays in the middle: it allows frame transition with internal timing, but only when transition is triggered by an external trigger. In this case, the Picture Time is the interval between two refresh of all mirrors: if external trigger is on, during refresh there will be a frame transition, while if the external trigger is off, after putting mirrors to the rest stat $\complement^{4}$, the DMD keeps repeating the last image before trigger switches off (see Fig. 2.13 below). Maximum value for the Picture is $44 \mu$ s. There are many options that can be set: in the ProjControl function, with ALP_LEVEL_HIGH (LOW) the frame transition is triggered by an high (low) level of the trigger signal, while with ALP_LEVEL_RISING (FALLING) the frame transition is triggered by the rising (falling) of the trigger signal.

### 2.2.4 DMD focusing procedure

I've already described the method we used to put the objective in focus on the atoms cloud, but in any case the cigar cloud has a width of about 25 mum , much higher than the depth of field of our objective (about $2 \mu \mathrm{~m}$ ); so, the focusing of the objective is

[^11]

Figure 2.14: Images acquired with high resolution imaging of the writing "Thank you" sent as optical potential on the atoms with the DMD (repulsive green light is the outline of the writing): zoom on the word "you". The difference between the left and right image is a movement of about $10 \mu \mathrm{~m}$ of the objective: in the right image, the contrast is higher and the edges are sharper, thus the objective is better focused.
limited by the high cloud width.
When we added the DMD, we needed to put in focus the intensity profile generated using the new device, moving an optical element independent from the imaging path already set: this is why we put a translator together the second lens of the demagnifying telescope in the DMD optical path (see Sect. 2.2.1). To focus the DMD image onto the cloud, we illuminate the atoms with sharp edged images, because to find the focused configuration was simpler, watching the cloud density distribution (e.g. see Fig. 2.14).
To exploit a high-resolution objective, to be on focus is a necessary task: indeed, the depth of field ${ }^{5}$ is about $2.5 \mu \mathrm{~m}$ for our diffraction-limited imaging system, thus any position fluctuation on the micrometric scale can affect our in-focus condition. In particular, we have two main kinds of fluctuations putting out of focus our imaging system: the thermal fluctuations of the metal mounting, which is made in aluminium and thus have a variation of the order of some microns per meter per degree, and the fluctuation of the infrared trapping beams, which are fractions of the beam waists and thus on the scale of microns. These fluctuation require a daily optimization of the objective focus position with the motorized T-stage: moving the objective, we restore the focus both of the imaging and of the DMD, even if we are moving not only the objective, but also the iris and the tube, and in this way changing the optical path between the tube lens and the PBS where the two beams are mixed. This is an important feature of our imaging system, due to high magnification: the depth of focus ${ }^{6}$ indeed is proportional to depth of field times the magnification squared, so it's about 400 times longer, while, as shown in Fig. 2.14, a movement of only $3 \mu \mathrm{~m}$ makes a visible difference in the atoms cloud image; our imaging system is thus insensitive both to the tiny movements to adjust the

[^12]

Figure 2.15: Example of highly resolved images of our cloud with high resolution optical potentials. The first is a mBEC with a Gaussian barrier, the second and the third are a mBEC and a UFG respectively, with the outline of the writing "LENS".
objective position and to the fluctuations of the Andor CCD or of the second lens of the DMD path.
As discussed, the new installed imaging system allows us to produce high resolution optical potential in a highly resolved cloud: in Fig. 2.15 we are showing only few example of what we can now achieve. The first image is a Bose-Einstein condensate (mBEC) of ${ }^{6}$ Li molecules with a Gaussian profiled barrier (obtained using the DMD) with a width $\sigma \approx 1.5 \mu \mathrm{~m}$; in second and third image we illuminate the atoms with the outline of the word "LENS" a mBEC and a Unitary Fermi Gas (UFG) respectively.

## Chapter 3

## Dynamical manipulation of the phase profile of a fermionic superfluid

One of the most important features of a superfluid is the ability to flow without friction, and if the fluid velocity exceeds a critical value, to dissipate the flow through elementary excitations. Thus, the capability to control the motion of a superfluid can give a powerful tool to study its properties.
In this chapter, I discuss the implementation of tailored potential on a superfluid of pairs on the BEC side ( $k_{F} a \simeq 0.25$ ) of the ${ }^{6} \mathrm{Li}$ Feshbach resonance, to show the possibility to trigger a motion manipulating the phase of the system. In the first part, I present a simple theoretical frame of our method of phase manipulation. Then, I show the measurements we performed on the BEC superfluid using a phase-step profile, a phase gradient on the whole gas and a triangular pattern to illuminate the cloud. In the end, I discuss the heating effects we observed during this measurements, and some outlooks about phase manipulation in a BEC cloud.

### 3.1 Theoretical description of phase imprinting

Let's consider a gas of $N$ particles, whose evolution is set by an Hamiltonian $H_{0}$, and suppose to switch on a perturbation $V(x)$ i.e. green light from the DMD, which is spatially dependent on the $x$ coordinate, at time $t=0$. The many-body wavefunction of the system evolves in time, in the Schödinger picture, as

$$
\begin{equation*}
|\psi(t)\rangle=\exp \left[-\frac{i}{\hbar}\left(H_{0}+V(x)\right) t\right]|\psi(0)\rangle \tag{3.1}
\end{equation*}
$$

being $|\psi(t)\rangle$ the many-body wavefunction, and $x=\sum_{j=1}^{N} x_{j}$ the quantum operator of the $x$ coordinate. If the perturbation acts for a time $t$ short enough to neglect the term
$H_{0} \not{ }^{\prime}$, whereas the value $V(x)$ is large enough not to be neglected. In this conditions, Eq. (3.1) becomes

$$
\begin{equation*}
|\psi(t)\rangle \approx \exp \left[-\frac{i}{\hbar} V(x) t\right]|\psi(0)\rangle \tag{3.2}
\end{equation*}
$$

It's important to underline that the time evolution term is acting on all the particles, depending on $x=\sum_{j=1}^{N} x_{j}$.
We compute the probability current to study the motion due to the perturbation $V$. The probability current operator $J_{x, i}(\mathbf{r}, t)$ of the $i$-th particle along $x$ in position $\mathbf{r}$ at time $t$ can be defined as 34

$$
\begin{equation*}
J_{x, i}(\mathbf{r}, t)=\frac{1}{2 m_{i}}\left(p_{x, i} \delta\left(\mathbf{r}-\mathbf{r}_{i}\right)+\delta\left(\mathbf{r}-\mathbf{r}_{i}\right) p_{x, i}\right) \tag{3.3}
\end{equation*}
$$

where $p_{x, i}$ is the momentum operator of the $i$-th particle along $x$ direction, and $\mathbf{r}_{i}$ is the position operator of the $i$-th particle. The current for the whole many-particle system along $x$ direction is $J_{x}(\mathbf{r}, t)=\sum_{i=1}^{N} J_{x, i}(\mathbf{r}, t)$. Now, we compute the mean value of $J_{x, i}$ on the many-body state $|\psi(t)\rangle$ :

$$
\begin{equation*}
\left\langle J_{x, i}(\mathbf{r}, t)\right\rangle_{\psi(t)}=\frac{1}{2 m_{i}}\left(\langle\psi(t)| p_{x, i} \delta\left(\mathbf{r}-\mathbf{r}_{i}\right)|\psi(t)\rangle+\langle\psi(t)| \delta\left(\mathbf{r}-\mathbf{r}_{i}\right) p_{x, i}|\psi(t)\rangle\right) \tag{3.4}
\end{equation*}
$$

Expressing the right-hand side of Eq. (3.4) in the position basis $|\mathbf{r}\rangle$, inserting a resolution of the identity $\mathbb{I}=\int d_{3 N} r|\mathbf{r}\rangle\langle\mathbf{r}|$ between $p_{x, i}$ and $\delta\left(\mathbf{r}-\mathbf{r}_{i}\right)$, and remembering $p_{x, i}|\psi\rangle=\left|\psi_{1}, \cdots, p_{x, i} \psi_{i}, \cdots, \psi_{N}\right\rangle$ and using the normalization condition of the singleparticle wavefunction $\int d_{3} r\left|\psi_{j}(\mathbf{r})\right|^{2}=1$, we obtain

$$
\begin{equation*}
J_{x, i}(\mathbf{r}, t)=-\frac{i \hbar}{2 m_{i}}\left(\psi_{i}^{\star}(\mathbf{r}, t) \frac{\partial}{\partial x_{i}} \psi_{i}(\mathbf{r}, t)-\psi_{i}(\mathbf{r}, t) \frac{\partial}{\partial x_{i}} \psi_{i}^{\star}(\mathbf{r}, t)\right) \tag{3.5}
\end{equation*}
$$

For Eq. (3.2), we obtain the relation

$$
\begin{align*}
& J_{x, i}(\mathbf{r}, t)=-\frac{i \hbar}{2 m_{i}}\left[\psi_{i}^{\star}(\mathbf{r}, 0) \frac{\partial}{\partial x_{i}} \psi_{i}(\mathbf{r}, 0)-\psi_{i}(\mathbf{r}, 0) \frac{\partial}{\partial x_{i}} \psi_{i}^{\star}(\mathbf{r}, 0)+\right. \\
&\left.+\left|\psi_{i}(\mathbf{r}, 0)\right|^{2}\left(e^{\frac{i}{\hbar} V(x) t} \frac{\partial}{\partial x_{i}} e^{-\frac{i}{\hbar} V(x) t}-e^{-\frac{i}{\hbar} V(x) t} \frac{\partial}{\partial x_{i}} e^{\frac{i}{\hbar} V(x) t}\right)\right] \tag{3.6}
\end{align*}
$$

In Eq. (3.6), the terms in the first line represent the current at $t=0$ driven by the Hamiltonian $H_{0}$, while the second line terms are driven by the perturbation $V(x)$ switched on at $t=0$. We can calculate now the total probability current by summing over $i$ dividing the $H_{0}$ term and the $V(x)$ term

$$
\begin{equation*}
J_{x, H_{0}}(\mathbf{r}, t)=-\sum_{i=1}^{N} \frac{i \hbar}{2 m_{i}}\left[\psi_{i}^{\star}(\mathbf{r}, 0) \frac{\partial}{\partial x_{i}} \psi_{i}(\mathbf{r}, 0)-\psi_{i}(\mathbf{r}, 0) \frac{\partial}{\partial x_{i}} \psi_{i}^{\star}(\mathbf{r}, 0)\right] \tag{3.7}
\end{equation*}
$$

[^13]If we assume the unperturbed many-body state to be stationary, we can set this current to be zero: $J_{x, H_{0}}(\mathbf{r}, 0)=0$.
Now, we consider the term due to the perturbation ${ }^{2}$ from Eq. (3.6), and deriving the two terms it becomes

$$
\begin{equation*}
J_{x}(\mathbf{r}, t)=-\sum_{i=1}^{N} \frac{1}{m_{i}}\left|\psi_{i}(\mathbf{r}, 0)\right|^{2} \frac{\partial}{\partial x_{i}} V(x) t \tag{3.8}
\end{equation*}
$$

In case of identical particles, $m_{i}=m, \forall i$. Let us assume the perturbing potential to be linear in $x$ i.e. $V(x)=\beta x$, all the derived terms in Eq. (3.8) are equal and we obtain

$$
\begin{equation*}
J_{x, l i n}(\mathbf{r}, t)=\frac{1}{m} \rho(\mathbf{r}, 0) \beta t \tag{3.9}
\end{equation*}
$$

where $\rho(\mathbf{r}, 0)=\sum_{i=1}^{N}\left|\psi_{i}(\mathbf{r}, 0)\right|^{2}$ is the density of the many-particle system 34 .
Eq. (3.8) shows that we can induce a current in the many particle system by only adding a dephasing to each particle through the perturbation $V(x)$. In particular, we generated this potential illuminating the system with tailored intensity profiles using the DMD. The three types of intensity shape we applied were a step function, which generates a phase step in the cloud according to Eq. (3.2), a single linear gradient on the whole cloud, which triggers a rigid movement of the cloud center of mass according to Eq. (3.9), and two opposite linear gradients (a triangular pattern) on the two halves of the cloud, which accelerates the two parts of the cloud in opposite directions. Since the gas is trapped inside a harmonic potential, the current induced by these phase manipulations couples to the collective modes of the trapped gas: in particular, the center of mass motion excites the dipole mod $\underbrace{3}$, whereas the motion triggered by the triangular pattern excites the quadrupole or "breathing" mode $\left.{ }^{4} \mid 35,36\right]$.

### 3.2 Phase step on a mBEC

The first measurement that we performed exploiting DMD-generated profiles consisted in illuminating only half of the mBEC cloud with a constant intensity. The fully-condensed state wavefunction can be written as

$$
\begin{equation*}
\psi(\mathbf{r}, t)=\sqrt{|\psi(\mathbf{r}, t)|^{2}} e^{i \arg [\psi(\mathbf{r}, t)]}=\sqrt{n(\mathbf{r}, t)} e^{i \phi(\mathbf{r}, t)} \tag{3.10}
\end{equation*}
$$

i.e. the product of an amplitude $\sqrt{n(\mathbf{r}, t)}$ and a phase $\phi(\mathbf{r}, t)$, thanks to having all particles in ground state $[37]$. For Eq. $(3.2)$, illuminating the BEC cloud with a constant intensity profile, the effect is a modification of the phase of the whole wavefunction, which is not observable. Thus, we decided to illuminate only half of the cloud; on the other hand, it has been previously studied [37, 38] that a superfluid imprinted with a phase step is undergoing the creation of a defect, a so-called dark soliton, in the correspondence

[^14]

Figure 3.1: Barrier (above) and phase imprinting (below) images produced on the DMD and used to imprint the mBEC cloud, with the intensity profiles cut on the two axis drawn as solid lines on the images. The barrier profile in the tight direction was fitted with a Gaussian to determine the width, which is $w=1.47 \mu \mathrm{~m}$.
of the rapidly twisted phasc ${ }^{5}$.
To avoid the creation of excitation in the superfluid during phase imprinting, we raised a high power barrier which was dividing the cloud into two different superfluid reservoirs, with a depletion of density in the center. Furthermore, in this way, when imprinting phase only on one reservoir, we were performing interference measurement using the not imprinted part of the cloud as reference. It's also well-established that two condensates locked in phase (coherent matter waves), when expanding in free space are giving rise to an interference pattern analogous to the double slit experiment one [37, 39]. Lets consider two condensates with a Gaussian wavefunction i.e. non-interacting atomic cloud in the ground state of a harmonic oscillator, with $R_{0}$ as initial width, having number of particles respectively $N_{1}$ and $N_{2}$, with centres separated by a distance $d$ and left expanding in free space. It can be demonstrated that, if the effect of the atomic interactions and external potentials is negligible, for long times of flight ${ }^{6}$, the density profile could be written as [37]

$$
\begin{equation*}
n(\mathbf{r}, t)=\frac{N_{1}+N_{2}}{\left(\pi R_{t}^{2}\right)^{3 / 2}} \exp \left[-\frac{r^{2}}{R_{t}^{2}}\right]\left[1+\frac{2 \sqrt{N_{1} N_{2}}}{N_{1}+N_{2}} \cos \left(\frac{\hbar t}{m R_{0}^{2} R_{t}^{2}} \mathbf{r} \cdot \mathbf{d}+\varphi_{1}-\varphi_{2}\right)\right] \tag{3.11}
\end{equation*}
$$

where $\varphi_{i}$ is the phase of the $i$-th condensate and $R_{t}^{2}=R_{0}^{2}+\frac{\hbar t}{m R_{0}}$ is the width of the clouds at time $t$. In the experiments, the fringes visibility can be reduced by the presence of a not negligible thermal fraction and of interactions, which remove particles from the

[^15]condensed state by the elastic non-forward scattering [40].
In Fig. 3.1, is shown the profile of the barrier and the flat top profile loaded on the DMD to imprint a controlled phase. Some roughness is visible on both the barrier and the flat top profile: the effect of this roughness, reduced by closing the LP filter of Fig. 2.8 as discussed in Sect. 2.2.2, was giving rise to some heating effects on the gas (see Sect. 3.5


Figure 3.2: Above, image of the mBEC gas with the barrier immediately after the phase imprinting. In the middle, the interference pattern from two clouds left expanding freely for 20 ms . Below, the green circles are the density profile of the image above, summed over the vertical axis; in yellow, the fit of the profile using Eq. 3.12 as fitting function.


Figure 3.3: Imprinted phase in $\pi$ units data as function of imprinting time for a 200 mW (blue circles) and 250 mW (green squares); errorbars show the standard deviation of the mean, computed in circular statistics (at least ten points per point were taken). The solid lines are the respective linear fit of the data.
for details).
The barrier was risen 2.740 s before the end of the evaporation, to avoid to disconnect the two BEC clouds, then, 240 ms after the 300 ms sweep Feshbach magnetic field to 702 G , one part of the cloud was illuminated for a variable time $\Delta t$. After, the optical trap and the barrier ( $100 \mu \mathrm{~s}$ later) were switched off, and the two clouds were left expanding in free space for a time-of-flight interference measurement. Then, the state $|1\rangle$ was imaged by the high-resolution vertical imaging system, giving rise to the image of interference pattern (one typical example is shown in Fig. 3.2.
The two-dimensional density profile was fitted with a 2D Gaussian function plus a sinusoidal term, due to the interference:

$$
\begin{equation*}
f_{f i t}(x, y)=B \exp \left[-\frac{\left(x-m_{x}\right)^{2}}{s_{x}}-\frac{\left(y-m_{y}\right)^{2}}{s_{y}}\right]\left[1-C \sin \left(\frac{2 \pi}{\lambda}\left(x-m_{x}\right)+\varphi\right)\right] \tag{3.12}
\end{equation*}
$$

in the custom Python program used in laboratory for image analysis. The $\varphi$ corresponds to the phase difference between the two clouds: the value of the fitted parameter spans from $-\pi$ to $\pi$, so we needed circular statistic tools to compute the mean and the standard deviation of the data [41].
In Fig. 3.3, the results of the phase imprinting calibration measurements are shown, for two different powers of the green beam ${ }^{77}$ The phase shift of the left-half cloud is proportional to (see Eq. (3.2) the intensity $I$ of the imprinting beam and the time $\Delta t$

[^16]

Figure 3.4: Histogram of the angular distribution of data for 200 mW of green power and $\Delta t=0 \mu \mathrm{~s}$ (on the left) and $\Delta t=100 \mu \mathrm{~s}$ (on the right). The spreading of the data shows that the relative phase of the two condensates is fixed.
of the imprinting, thus the phase imprinting should change linearly with the imprinting time; furthermore, fitting with a straight line the data for different power, the ratio between powers should be the same of the ratio of the slope of the lines.
We fitted the data with a linear fit, to extract the slopes for both the powers. The results are:

$$
\begin{align*}
& \delta \varphi_{200}=(-2.32 \pm 0.09) \frac{\pi}{\mathrm{ms}} \\
& \delta \varphi_{250}=(-3.21 \pm 0.08) \frac{\pi}{\mathrm{ms}} \tag{3.13}
\end{align*}
$$

where errors are standard errors of the fit. I computed the ratio of these slopes, to be compared to the power ratio $250 / 200=1.25$ :

$$
\begin{equation*}
\frac{\delta \varphi_{250}}{\delta \varphi_{200}}=1.38 \pm 0.07 \tag{3.14}
\end{equation*}
$$

where the error is the square sum of the relative errors. The result is compatible with the expected value.
In Fig. 3.4 I show also a typical histogram of the angular distribution of the phase measurements, which shows the phase to be fixed, not spreading over the whole interval $[-\pi, \pi]$. This was an important features to check: raising the barrier during last stages of the evaporation, indeed, when the gas is already condensed and thus phase coherent, is not an insurance to not disconnect the two condensates, letting them acquire a random relative phase.


Figure 3.5: Linear gradient image loaded on the DMD and used to imprint the mBEC cloud, with the intensity profiles cut on the two axis drawn as solid lines on the images.

### 3.3 Single phase gradient

Next, we used a linearly varying light shift along the axial direction to induce an axial current in the system: according to Eq. (3.9), indeed, with a linear gradient of intensity in the $x$ direction, which produces a linear gradient perturbation, the cloud after being illuminated should move with a velocity

$$
\begin{equation*}
u_{x}(\mathbf{r}, t)=\frac{1}{\rho(\mathbf{r}, 0)} J_{x}(\mathbf{r}, t)=\frac{1}{m} \beta \Delta t \tag{3.15}
\end{equation*}
$$

Thus, the velocity imprinted has to be proportional to the intensity gradient illuminating the cloud. We performed a measurement to demonstrate the light shift to be able to imprint a velocity to the BEC and to calibrate the range of velocities we can excite with our DMD system.
After a mBEC with at least $85 \%$ of condensed fraction at 702 G of Feshbach field was


Figure 3.6: Above, an image of the BEC cloud immediately after the phase gradient imprinting ( $t=0 \mathrm{~ms}$ ). Below, the cloud at $t=20 \mathrm{~ms}$ after the imprinting: the cloud has visibly moved to the right.
produced, we illuminated the cloud with a gradient intensity profile loaded on the DMD for a variable time $\Delta t$, shown in Fig. 3.5, 500 mirrors long and 200 mirrors wide, to cover the whole cloud. The green beam power ${ }^{8}$ was 650 mW . Fig. 3.6 shows the movement excited by the phase gradient imprinting. By fitting the position of cloud centre exactly at the end of the illumination with the green light and after $\bar{t}=10 \mathrm{~ms}$ of evolution, we extracted the velocity by the ratio

$$
\begin{equation*}
v=\frac{\Delta x}{\bar{t}} \tag{3.16}
\end{equation*}
$$

In Fig. 3.7 the results for the mBEC cloud are shown, together with the results of the fit I performed using the function

$$
\begin{equation*}
v(\Delta t)=v_{0}+a_{i m p r} \cdot \Delta t \tag{3.17}
\end{equation*}
$$

Fit results of velocity depending on imprinting time are:

$$
\begin{equation*}
v_{B E C}=\left(0.4 \pm 0.1 \frac{\mathrm{~mm}}{\mathrm{~s}}\right)+\left(4.9 \pm 0.5 \frac{\mathrm{~mm}}{\mathrm{~s} \cdot \mathrm{~ms}}\right) \Delta t \tag{3.18}
\end{equation*}
$$

where the errors are the standard errors of the fit. Interestingly, these measurements shows that with our system is possible, in principle, to excite a velocity close to the critical velocity of the mBEC, which is on the order of few $\frac{\mathrm{mm}}{\mathrm{s}} \sqrt[42]{ }$. Thus using this technique can be used to observe and study the flow of a superfluid in desired geometry,


Figure 3.7: Velocity as function of imprinting time for the BEC cloud. Data are blue circles, where error bars show standard deviation of the mean (at least five points per point were taken), while green solid line is the fit.

[^17]

Figure 3.8: Double linear gradient image loaded on the DMD and used to imprint the mBEC cloud, with the intensity profiles cut on the two axis drawn as solid lines on the images.
shear viscosity effects or transport through channels and obstacles in the condensates. The same measurement was also performed on a thermal cloud (condensed fraction lower than $10 \%$ ), to confirm the velocity imprinting protocol: Eq. 3.9 indeed holds also for non-condensed gases. However, we needed a smaller cloud to cover it fully with the DMD pattern, having thus a lower imaging signal: although this could have affected the density profile fit, giving rise to a lower measured acceleration, it was possible to trigger the same dynamics also in the thermal gas.

### 3.4 Double phase gradient

Next, we used a triangular pattern, with two linearly varying light shift with respect to the center of the cloud, as shown in Fig. 3.8. For Eq. (3.8), we were in this case exciting two opposite currents, thus coupling mainly to the quadrupole mode of the trap.


Figure 3.9: Above, image of the BEC cloud immediately after the imprinting of the triangular pattern $(\Delta t=50 \mu \mathrm{~s}, P=600 \mathrm{~mW})$. Below, image of the cloud 20 ms after the imprinting: the enlargement due to the motion excited by the light shift is visible.


Figure 3.10: Oscillation of the cloud width for an imprinting time $\Delta t=75 \mu \mathrm{~s}$ and a green power of 400 mW . Errorbars are standard deviation of the mean (at least five points per point were taken). The solid line is the sinusoidal fit performed on the point until the first minimum, while the shaded regions are the $95 \%$ confidence bands. The fitted frequency error was clearly strongly affected by the lack of at least an entire period to be fitted.

As a consequence, the gas is expected trigger a relative motion of the two halves in opposite directions, and thus to have an increasing in the width. To have a qualitative understanding of the motion, we extracted the information about the dynamics from a Gaussian fit of the density profile of the gas.
Let's consider a Gaussian cloud with width $\sigma_{0}$ in the centre of a harmonic potential: we model the acting of the double gradient as increasing of the width of the Gaussian distribution. The most far position reached by a particle in positions $x=\frac{\sigma_{0}}{2}$, with a starting velocity $v_{0}$ and moving away from the centre of the harmonic potential, is

$$
\begin{equation*}
x_{F}=\sqrt{\frac{\sigma_{0}^{2}}{4}+\frac{v_{0}^{2}}{\omega^{2}}} \tag{3.19}
\end{equation*}
$$

where $\omega$ is the harmonic frequency of the potential. Thus, considering the two halves of the cloud moved by the double gradient pattern, we can estimate the maximum expected width of the cloud as:

$$
\begin{equation*}
\sigma_{M} \approx x_{l e f t, F}-x_{r i g h t, F}=\sqrt{\sigma_{0}+4 \frac{v_{0}}{\omega}} \tag{3.20}
\end{equation*}
$$

Inverting this relation, from the width of the cloud Gaussian fit, we could estimate the velocity imprinted on the halves of the cloud.

The measurement was performed imprinting the same BEC cloud as previous sections, illuminating the gas with the triangular pattern at power $P$ for a time $\Delta t$. As explained in Sect. 3.2, the imprinted phase is proportional to the product $P \cdot \Delta t$, thus, to have a constant imprinted phase, we reduced the green power while increasing the imprinting time. Then, the imprinted velocity was expected to be constant, at least since the imprinting time is lower than the typical time scales of the unperturbed Hamiltonian (see Sect. 3.2): this time is required to propagate the perturbation through the system, and is proportional to the inverse of typical excitation energies. The many-body excitations (in a BEC, sound) set an upper bound to energy at the chemical potential $\mu$, while a lower bound is set by the energies of the trap modes. Thus the velocity should begin to depend on the imprinting time $\Delta t$ and not only on the light shift gradient when $\hbar / \mu \lesssim \Delta t \lesssim 1 / 2 \omega_{x}$, where $\omega_{x}$ is the axial trap frequency and $2 \omega_{x}$ is the frequency of the quadrupole mode: indeed only trap modes with the same symmetry of the excitation can be excited, and the breathing mode is the lowest energetic one.
The initial width, the maximum width and the oscillation frequency used to estimate the velocity were extracted from the fit of the first period points of the oscillation: indeed, as it visible for example in Fig. 3.10, when the two halves of the cloud were colliding after the initial excitation, the cloud width was remaining more or less constant for 10 ms , then the cloud was showing a usual breathing mode. Thus, we couldn't fit the whole oscillation, and we assumed that the dynamics connected to response time of the system is more visible in first oscillation, before the rearrangements between the two clouds after the first collision; this choice was slightly increasing the error on the fitted frequency. The


Figure 3.11: Velocity estimation as a function of the imprinting time at constant imprinted velocity. The error bars are obtained by errors propagation from the standard errors of the fit for each imprinting time.
difference in oscillation amplitude was in any case visible also after the collision. The slowly decrease in the mean width, on the other hand, is probably connected to the atoms number decrease in the cloud.
In Fig. 3.11 the results of the estimated velocity as function of the imprinting time are shown. The interesting feature is the strong difference in the velocity when the time is $150 \mu \mathrm{~s}$ : the last two point indeed are showing a lower velocity, while the imprinted phase is the same of the previous. We attributed this behaviour to the imprinting time to be comparable to the response time of the system: with a rough estimation, for our trap frequencies and a typical number of atoms of $10^{5}$, the many-body response time scale is $t_{m b} \sim \frac{\hbar}{E_{F}} \approx 180 \mu \mathrm{~s}$.

### 3.5 Roughness in intensity profile

The images of the intensity profile shown in the previous sections are showing some roughness in the profile. According to Eq. (3.8), this roughness is generating phase then velocity fluctuations in the imprinted cloud. While, as said in Sect. 2.2.2, we could reduce the effective roughness by closing the filtering iris, it was also interesting to study the effect of these velocity fluctuations on the atoms.


Figure 3.12: Condensed fraction as function of time for different imprinting times and different iris aperture. Blue circles are zero imprinting time, as reference. Green squares are $100 \mu$ s of imprinting time, and yellow diamonds are $200 \mu \mathrm{~s}$, both with iris slightly open. Red upper triangles and orange lower triangles are $100 \mu \mathrm{~s}$ and $200 \mu \mathrm{~s}$ respectively, with iris closed. When the iris is closed, the decrease in condensed is clearly smaller. All errorbars are standard deviation of the mean.


Figure 3.13: mBEC cloud after $200 \mu$ s of phase imprinting at 600 mW of power.

To this purpose, we took some images applying a phase imprinting on a mBEC cloud with the iris partially opened and fully closed, and studied the effects. The noticeable consequence of opening the iris was a heating of the cloud, as shown through the condensed fraction decrease in Fig. 3.12,
The heating effect of the roughness of the intensity profile is probably due to the conversion of the velocity fluctuations to density fluctuations, through collisions among atoms; the density is coupling together with excitations in the cloud, like vortices and Bogoliubov modes (sound-like waves).
Furthermore, after illuminating at high power ( 600 mW of our green beam) the cloud for some hundreds of microsecond, the gas was showing lots of excitation in the density profile, as visible for example in Fig. 3.13.

### 3.6 Outlooks of phase manipulation

The measurements described in this chapter open the route to a wide variety of experiments of phase manipulation. Firstly, as shown in Sect. 3.2, I demonstrated that an arbitrary phase profile can be imprinted on the cloud, thus we can create phase domains in the gas.


Figure 3.14: Examples of future experiments which can be performed thanks to phase manipulation.

Furthermore, Sect. 3.3 and Sect. 3.4 show we are also able to trigger a controlled dynamics through intensity gradients: thus, in principle, is possible, for example in a toroidal geometry, to study the rotating motion of the degenerate gas, or, exciting velocities close to the critical one, also Kelvin-Helmoltz instability, as proposed in 43].
Moreover, we can also create an obstacles pattern and accelerate the degenerate gas close to critical velocity against the obstacles, to study transport phenomena.
As last example, also the preliminary measurement shown in Sect. 3.5 are very interesting, because I demonstrated the DMD to be capable to heat the gas. This can be used to heat in controlled way the cloud to study second sound, which is connected to heat transfer in superfluid phase and about which lot of open questions still remain [44].

## Chapter 4

## Towards 2-dimensional Fermi gases

One of the main goals in the implementation of high-resolution imaging and optical potential is the experimental investigation of ultracold quasi-two-dimensional Fermi gas. During this thesis project, I also developed an optical set-up to realize a suitable single plane quasi-2D confinement, with a $\mathrm{TEM}_{01}$-like green beam. In this chapter, I will first discuss some features of 2D Fermi gases; then, the steps to design the optical dipole trap will be presented. Then, I will show the implementation and characterization of the beam shape and the plan to implement the beam on the atoms, with some other necessary modifications in the experiment set-up to produce and study a quasi-2D gas.

### 4.1 2D ideal Fermi gases in harmonic traps

Going from three to two dimensions, the fundamental quantities describing a degenerate gas in a harmonic trap are modified. The 2-dimensional trapping potential can be defined as:

$$
\begin{equation*}
V(x, y)=\frac{1}{2} m\left(\omega_{x} x^{2}+\omega_{y} y^{2}\right) \tag{4.1}
\end{equation*}
$$

where $m$ is atomic mass and $\omega_{i}$ are the trapping frequencies. The corresponding density of states in 2D is:

$$
\begin{equation*}
g(\epsilon)=\frac{\epsilon}{2 \hbar^{2} \omega_{r}^{2}} \tag{4.2}
\end{equation*}
$$

where $\omega_{r}=\left(\omega_{x} \omega_{y}\right)^{1 / 2}$ is the geometric mean trap frequency. Through the density of states, as done in Sect. 1.2.2, and the distribution function, we can obtain the Fermi energy $E_{F}$ for the system, at fixed number of fermions $N$ :

$$
\begin{equation*}
E_{F}=\sqrt{2 N} \hbar \omega_{r} \tag{4.3}
\end{equation*}
$$

The density profile of the trapped gas is computed using the local density approximation (LDA), in which the sample density distribution is locally approximated by a uniform one. At zero temperature, the density can be written as:

$$
\begin{equation*}
n_{F}(\mathbf{r}, T=0)=\frac{m}{2 \pi \hbar^{2}}\left(E_{F}-V(\mathbf{r})\right) \tag{4.4}
\end{equation*}
$$

From Eq. (4.4), we can extract the Thomas-Fermi radius $R_{i, F}=\sqrt{2 k_{B} T_{F} /\left(m \omega_{i}^{2}\right)}$, i.e. the maximum cloud size given by the relation $V\left(R_{i, F}\right)=E_{F}$, replacing $k_{B} T_{F}=E_{F}$ :

$$
\begin{equation*}
R_{i, F}=(8 N)^{\frac{1}{4}} \frac{\omega_{r}}{\omega_{i}} \sqrt{\frac{\hbar}{m \omega_{r}}} \tag{4.5}
\end{equation*}
$$

This quantities characterizing a non-interacting Fermi gas at zero temperature, are actually useful for an estimation of the parameters needed to create a quasi-2D gas.

### 4.1.1 Quasi-2D confinement

The realization of a quasi-2D gas is typically achieved by strongly confining a 3D cloud in the axial direction of the harmonic trap. In this conditions, the dynamic on the axial direction is "frozen out" by means of the tight confinement i.e. all atoms are in the ground state of the axial oscillator. On the other hand, if the scattering length $a$ is comparable or lower than the oscillator width $l_{z}=\sqrt{\hbar / m \omega_{z}}$, the interactions between atoms are the same of the 3D system. Thus, an interacting degenerate gas behaviour can deviate significantly from theoretical prediction of the non-interacting case e.g. the transverse width of a cloud could not saturate lowering the atom number [45]. for this reason, this kind of gases are called "quasi-2D".
Due to the high anisotropy of the trap, the quantum gas is expected to populate oscillator levels in two dimensions only and thermally excited particles cannot move along the tightly confined direction. We find the fermionic gas in this condition of quasi-2 dimensionality if the criterion:

$$
\begin{equation*}
\mu<\hbar \omega_{z} \tag{4.6}
\end{equation*}
$$

is fulfilled, where $\mu$ is the chemical potential. At zero temperature, Eq. (4.6 becomes simply $E_{F}<\hbar \omega_{z}$, whereas at high temperatures can be written as $k_{B} T \ll \hbar \omega_{z}$. In


Figure 4.1: Illustration of the criterion for 2D gases. In green are sketched the radial oscillator levels, while in light blue the axial ones. Both the Fermi energy $E_{F}$ and the chemical potential $\mu$ have to be lower than the spacing between two levels of the axial harmonic oscillator $\hbar \omega_{z}$. The state of the radial direction, with spacing $\hbar \omega_{r}$, have to be populated, whereas only the ground state is populated along the axial direction.

Fig. 4.1 we sketched the scheme of the radial and axial harmonic oscillator levels, together with the criterion for quasi-2D confinement.
The quasi-2D condition, which is analogous to request the atom number in the excited states is much lower than in the ground state, sets a maximum number of atoms that can populate the system: one can estimate this number for a non-interacting Fermi gas by counting the number of available levels of the radial harmonic oscillator with energy lower than the spacing of the levels in the axial direction. The energy spectrum of the lowest energy state, with only axial ground state occupied i.e. $n_{z}=0$, can be written as:

$$
\begin{equation*}
E_{g s, 2 D}=\hbar \omega_{x}\left(n_{x}+\frac{1}{2}\right)+\hbar \omega_{y}\left(n_{y}+\frac{1}{2}\right)+\frac{1}{2} \hbar \omega_{z} \tag{4.7}
\end{equation*}
$$

In an ideal Fermi gas this energy must be lower than of the energy of the first excited state in the axial direction, given by $n_{z}=1, n_{x}=n_{y}=0$ :

$$
\begin{equation*}
E_{f e}=\frac{1}{2} \hbar \omega_{x}+\frac{1}{2} \hbar \omega_{y}+\frac{3}{2} \hbar \omega_{z} \tag{4.8}
\end{equation*}
$$

Requesting $E_{g s, 2 D}<E_{f e}$, we obtain a relation among the harmonic frequencies and the occupation numbers of the radial states:

$$
\begin{equation*}
n_{x} \omega_{x}+n_{y} \omega_{y}<\omega_{z} \tag{4.9}
\end{equation*}
$$

For $\omega_{x} \approx \omega_{y} \equiv \omega_{r}$, we define the trap aspect ratio as $A=\omega_{z} / \omega_{r}$ and the radial occupation number $n_{r}=n_{x}+n_{y}$. Using the condition in Eq. (4.9) and considering the degeneracy of harmonic oscillator states with $n_{x}+n_{y}=n_{x}^{\prime}+n_{y}^{\prime}$, we can approximate the critical number of atoms for the quasi-2D gas:

$$
\begin{equation*}
N_{\text {crit }, 2 D}=\sum_{n_{r}=0}^{\lfloor A-1\rfloor} n_{r}+1=\frac{\lfloor A\rfloor^{2}+\lfloor A\rfloor}{2} \tag{4.10}
\end{equation*}
$$

Thus, the number of atoms we can load in a quasi-2D trap does not depend on the absolute value of the trapping frequencies, but only on their ratio. For example, trapping frequencies of e.g. $\omega_{r}=2 \pi \cdot 30 \mathrm{~Hz}$ and $\omega_{z}=2 \pi \cdot 6 \mathrm{kHz}$ provide us an aspect ratio of 1:1:200 and the maximum number of fermions we can load in the quasi-2D harmonic trap is about $N_{\text {crit }, 2 D} \simeq 2 \cdot 10^{4}$.
Obviously, this rough calculation is accurate only in the case of non-interacting or weaklyinteracting Fermi gas, which we can generate in our experiment exploiting the broad Feshbach resonance of the ${ }^{6} \mathrm{Li}$ atoms in the ground state. At the so-called "zero-crossing", where the 3D scattering lenght is approaching zerd the degenerate gas behaves as an ideal Fermi gas. When, on the other hand, the magnetic field is closer to the Feshbach resonance field, the behaviour of the gas is more complicated and similarly the criterion to have 2D kinematics [47].

[^18]
### 4.2 Designing the optical trap

To generate a sufficiently tight potential, we have chosen a configuration already used in Ref. [48, 49] i.e. a $\mathrm{TEM}_{01}$-like blue-detuned intensity profile. This can be obtained by means of a $\pi$ step-phase plate acting on a Gaussian beam, which add a phase of $\pi$ only to half of the beam, as sketched in Fig. 4.2. In this way, we are able to trap a single-plane quasi-2D Fermi gas. The $\pi$ phase plate is manufactured by Silios Technologies, made in fused silica for 532 nm , with a refracting index of 1.4607 and an etched depth of 577 nm ; the active region is half of a $2^{\prime \prime}$ diameter disk.

### 4.2.1 Theoretical intensity shape

The electric field of a transverse electric mode with radial and axial mode number $l$ and $m$ respectively, for a beam propagating along the $y$ axis, is given by 50]

$$
\begin{align*}
E_{l m}(x, y, z)= & A_{l, m} \frac{W_{0}}{W_{x z}(y)} H_{l}\left(\frac{\sqrt{2} x}{W_{x}(y)}\right) H_{m}\left(\frac{\sqrt{2} z}{W_{z}(y)}\right) \times \\
& \times \exp \left[-i k y-i \frac{x^{2}+z^{2}}{2 R(y)}+i(l+m+1) \zeta(y)\right] \tag{4.11}
\end{align*}
$$

where $A_{l, m}$ is the amplitude of the electric field, $W_{x z}=\left(W_{x} \cdot W_{z}\right)^{1 / 2}$ and $W_{i}(y)=$ $W_{0 i} \sqrt{1+\left(y / y_{R}\right)^{2}}$ is the beam waist in the $i$ direction, $H_{l}(x)$ is the $l$-th Hermite-Gauss function, $R(y)$ is the curvature of the beam, $\zeta(x)=\arctan \left(\frac{y}{y_{R}}\right)$ and $y_{R}$ is the Rayleigh


Figure 4.2: A scheme of the Gaussian beam incoming on the $\pi$ phase plate. On the right, the profile of the electric field $E$ along the $z$ axis on the phase plate is showed.
range of the beam. The corresponding intensity distribution from the Eq. (4.11) is

$$
\begin{equation*}
I_{l m}(x, y, z)=\left|A_{l, m}\right|^{2}\left(\frac{W_{0}}{W_{x z}(y)}\right)^{2} H_{l}^{2}\left(\frac{\sqrt{2} x}{W_{x}(y)}\right) H_{m}^{2}\left(\frac{\sqrt{2} z}{W_{z}(y)}\right) \tag{4.12}
\end{equation*}
$$

The intensity profile for the mode $\mathrm{TEM}_{01}$ is the one we ideally want for our trap, and can be extracted from Eq. 4.12 by setting the mode indices to $l=0$ and $m=1$ :

$$
\begin{equation*}
I_{01}(x, y, z)=\frac{8 P z^{2}}{W_{x}(y) W_{z}^{3}(y)} \exp \left(-\frac{-2 x^{2}}{W_{x}^{2}(y)}-\frac{-2 z^{2}}{W_{z}^{2}(y)}\right) \tag{4.13}
\end{equation*}
$$

where $P$ is the beam power. Ideally, this is the best solution to produce the our trapping potential; unfortunately, we don't have such a laser source in our laboratory and, moreover, the required power make inconvenient to build a cavity on purpose.
To generate a similar profile, we use a Gaussian $\mathrm{TEM}_{00}$ beam passing through the $\pi$ phase plate focused on the atoms. The phase plate introduces a phase retardation of $\pi$ on half of the beam along the $z$ axis, so the field in the negative side of the $z$ axis is inverted, as shown in Fig. 4.2 (we choose to put $y=0$ plane in correspondence of the phase plate):

$$
\begin{equation*}
E_{p p}(x, 0, z)=E_{00} \times[\theta(z)-\theta(-z)] \tag{4.14}
\end{equation*}
$$

The $\pi$ phase plate removes the pure Gaussian mode from the beam, such that the field is now a superposition of higher TEM modes; performing a Fourier transform of this field through a lens, one can obtain the field expression in the focal plane ( $f$ is the focal length of the lens):

$$
\begin{equation*}
E_{p p}(x, f, z)=-i A_{00} \frac{1}{\sqrt{\tilde{W}_{0 x} \tilde{W}_{0 z}}} \exp \left(-\frac{x^{2}}{\tilde{W}_{0 x}^{2}}-\frac{z^{2}}{\tilde{W}_{0 z}^{2}}\right) \operatorname{erfi}\left(\frac{z}{\tilde{W}_{0 z}}\right) \tag{4.15}
\end{equation*}
$$

where $\tilde{W}_{0 i}=\lambda f /\left(\pi W_{0 i}\right)$ is the Fourier-transformed beam waist (equivalent waist). The erfi $(x) \equiv-i \operatorname{erf}(i x)$ function is called imaginary error function and it's real for real


Figure 4.3: Comparison between $\mathrm{TEM}_{01}$ and $\pi$ phase plate intensity profiles. On the left is showed the intensity of both the beams as function of $z$ coordinate. On the right there are the intensity distributions. Already at a first sight, the power of the phase plate beam appears to be distributed on a wider area, thus to generate a less efficient confinement.
values of $x$.
Then, one can compute the intensity shape for the beam from Eq. 4.15; as in Eq. 4.13, considering a beam power $P$, the intensity is

$$
\begin{equation*}
I_{p p}(x, f, z)=\frac{2 P}{\pi \tilde{W}_{0 x} \tilde{W}_{0 z}} \exp \left(-\frac{2 x^{2}}{\tilde{W}_{0 x}^{2}}-\frac{2 z^{2}}{\tilde{W}_{0 z}^{2}}\right)\left[\operatorname{erfi}\left(\frac{z}{\tilde{W}_{0 z}}\right)\right]^{2} \tag{4.16}
\end{equation*}
$$

where the $2 / \pi$ is a normalization factor. The intensity shape can be approximated in the vicinity of the central minimum as [51]

$$
\begin{equation*}
I\left(x=0, z \ll \tilde{W}_{0 z}\right) \simeq \frac{8 P}{\pi^{2} \tilde{W}_{0 x} \tilde{W}_{0 z}}\left(\frac{z}{\tilde{W}_{0 z}}\right)^{2} \tag{4.17}
\end{equation*}
$$

The final intensity shape obtained through our configuration is quite different from the usual $\mathrm{TEM}_{01}$ one, as is shown in Fig. 4.3. for the same power, the intensity of the beam obtained through the phase plate is spread over a wider area, producing in this way a weaker confinement. Thanks to explicit expressions for both the beams, one can compute the ratio between the trapping frequencies in the $z$ direction:

$$
\begin{equation*}
\frac{\omega_{z, P P}}{\omega_{z, T E M}}=\frac{1}{\pi} \tag{4.18}
\end{equation*}
$$

Although is less confining, for sake of simplicity in our optical path, we decided to use the $\pi$ phase plate alone, instead of adding elements to have a more $\mathrm{TEM}_{01}$-like intensity profile (see Ref. [48]).

### 4.2.2 The optical path

Before designing the set-up for the quasi-2D confining beam, I performed a study of the whole optical trap to extract the most suitable beam parameters. Indeed, while on the $z$ axis, the beam has a confining effect, in the $x y$ plane is slightly anti-confining: we need thus a confining potential in the plane. In our experiment, the Feshbach field has a slightly confining effect in the plane, $\omega_{F} \simeq 2 \pi \times 8 \mathrm{~Hz}$, but we need a more confining potential, to avoid too large clouds exceed the field of view of our objective. Thus, we added a vertical infrared Gaussian beam. Studying the trap shape for this potentials, I computed the parameters, compatible with our available hardware, for both the two confining beams (infrared and green): the green waist had to be around $\tilde{W}_{0 z} \times \tilde{W}_{0 x}=7 \times 430 \mu \mathrm{~m}$, with a green power of 2.4 W at 532 nm wavelength, while the infrared Gaussian waist had to be around $300 \mu \mathrm{~m}$ wide with a power of 1.2 W ; these parameters leaded to next trap frequencies:

$$
\begin{equation*}
\omega_{x}=\omega_{y} \simeq 2 \pi \times 29.4 \mathrm{~Hz} \quad \omega_{z} \simeq 2 \pi \times 7.5 \mathrm{kHz} \tag{4.19}
\end{equation*}
$$

In this conditions, we expect an ideal Fermi gas with at maximum $N_{\text {crit, } 2 D} \simeq 3.3 \times 10^{4}$ atoms. In case we load the $75 \%$ of $N_{\text {crit }, 2 D}$, we obtain a gas with $E_{F, 2 D}=0.31 \mu \mathrm{~K}$ and Thomas-Fermi radius in the $x y$ plane of $160 \mu \mathrm{~m}$.
In Fig. 4.4 the optical path used to characterize the green beam is sketched: the source


Figure 4.4: Optical path, seen from above, to characterize the profile of the trapping beam.
The focus $f_{1}$ of the last lens was changed to provide different magnifications measuring the $z$ direction waist.
was initially a 532 nm laser diode, then, I used the Verdi V-8 laser, to have a more monochromatic source. The beam is going through a Shäfter-Kirchhoff fibre collimator (60FC-T-4-M30-01), and through a half-wave plate to a high-power polarized beamsplitter (HPBS), to clean the polarization of the beam. The reflected beam is the one going away in the path, to a cylindrical lens, with $f=-200 \mathrm{~mm}$ in the vertical axis, so is acting only on the $z$ axis. This lens is rigidly connected to a dichroic mirror (DM) and to a spherical lens $f=500 \mathrm{~mm}$, and a mirror. The pair of lenses on the vertical axis performs a telescope on the beam, with expected magnification $M_{1}=2.5$, while only the second lens acts on the horizontal axis of the beam, focusing the beam. The lenses were chosen after a Mathematica simulation of the propagation of the laser beam, shown in Fig. 4.5, to get the necessary lens fot the waist required dimensions. The spherical lens position can be finely set by a translator with respect to the cylindrical lens.
Then there is another mirror, for having two before the $\pi$ phase plate for the alignment. The beam is going through the phase plate and, with another pair of mirrors, into the $f=150 \mathrm{~mm}$ spherical lens: this lens establishes a telescope on the horizontal axis of the beam, with magnification $M_{2}=0.3$, while is focusing the beam on the vertical direction. The intensity profile in the focus of this lens is the one we are going to use to trap the atoms. For this reason, to be very precise in the position of this lens, the lens is mounted on a translation stage along the optical axis, and is centred by the use of a target; furthermore, the lens is in a gimbal mount, to correct any angle misalignment between the lens surface and the beam. The choice of the focus of the last lens is fixed by the fact that we are entering in the science chamber through the same window of the horizontal imaging, which is using a $f=150 \mathrm{~mm}$ lens as last lens. Thus, using the same


Figure 4.5: Mathematica simulation of the beam propagation in our optical path. Red solid lines denote lenses positions, yellow one denotes the phase plate position, green one indicates the atom cloud. Light blue solid lines are the waist evolution along $x$ and $z$ axes.
lens we are reducing the modification needed to implement the new optical potential. The intensity profile is imaged with a Thorlabs CCD DCC15445M, with a sensor made of $1280 \times 1024$ pixels $5.2 \mu \mathrm{~m}$ large. To see the profile in the wide dimension I acquired images without any magnification, while to see the profile in the tight one, I needed a high magnification: to this purpose, I used a Mitutoyo objective G Plan Apo $20 \times$, infinity corrected, with working distance $w_{d}=29.42 \mathrm{~mm}$ and focal length $f=10 \mathrm{~mm}$; as tube lens I use two different achromatic lenses with $f=150 \mathrm{~mm}$ and $f=300 \mathrm{~mm}$, to have enough magnification to resolve the details in the intensity shape. Actually, the scales of the two waists are really different: by Mathematica simulations, the largest waist is expected to be $430 \mu \mathrm{~m}$ large, whereas the tightest one only $7 \mu \mathrm{~m}$; thus, we have a factor of 60 between the two scales.
After taking the measurement, I added a window perfectly similar to the ones of the science chamber, to check the effects on the green beam shape and position. The only difference was the position along the optical axis of the beam focus, because of the different optical path, but the shape and the position in plane weren't affected.

### 4.2.3 Preliminary measurements

To characterize the profile of the green confining beam, I measured the beam waist out of the collimator. In this way, I had fundamental information to build the two telescopes, compatible with the available space in the experiment and with the requested confinement of the optical potential. As a first rough estimation, I simply acquired an image of the beam waist and fit it with a radial Gaussian function, defined as $A+$ $B \exp \left[-2 \frac{\left(x-m_{x}\right)^{2}+\left(y-m_{y}\right)^{2}}{w^{2}}\right]$. The data and the fit are showed in Fig. 4.6 the waist measured is $w=1.391 \mathrm{~mm}$. This measure has been performed only to have a reference for the initial beam waist, to put in the Mathematica program.
To have a more precise measure of the beam waist, I performed a sequence of measures of the collimated beam divergence, and fitted with the function describing the evolution of the waist (with $y$ the propagation axis):

$$
\begin{equation*}
W(y)=W_{0} \sqrt{1+\left(\frac{y}{y_{R}}\right)} \tag{4.20}
\end{equation*}
$$

where $W_{0}$ is what is usually called beam waist, and $y_{R}=\frac{\pi W_{0}^{2}}{\lambda}$ is the Rayleigh range. Thus, having measures for different positions along the axis of beam propagation and knowing the wavelength of the laser, I extracted the beam waist with a one-parameter fit. In Fig. 4.7 is showed the result: all the images were fitted by a 2D Gaussian,


Figure 4.6: On the left, the image acquired by the CCD. On the right, the cuts on $x$ and $y$ axis with the fit performed onto 2 D profile and projected on the respective axis. Data are light blue solid line and fit is green solid line.


Figure 4.7: Plot of the waist measurements as function of distance from the fibre collimator. The shaded region represents the $95 \%$ confidence level of the fitted function.
with two different waists on the two axis, to verify the goodness of azimuthal symmetry assumption; then, because in all cases the two waist were compatible, I fitted all of them with a 2D radial Gaussian and extracted the results showed. The value of the waist is:

$$
\begin{equation*}
W_{0}=1.437 \pm 0.007 \mathrm{~mm} \tag{4.21}
\end{equation*}
$$

where error is the standard error of the fit.
After the measure of the waist, I aligned the beam in the first part of the path, until the mirror after the $f=500$ lens; then, watching by eye to the beam collimation in the vertical axis (the only one collimated, in the other direction the beam is focusing), I moved the spherical lens to conjugate it to the cylindrical one. After adding all the mirrors, I put the final $f=150$ spherical lens, the window and the system to magnify the beam.
With this system, as first thing I calibrated the magnification of the imaging system with both the tube lenses, in two ways: by moving an obstacle of a known distance and measuring the movement observed in the CCD, and by using a pinhole $100 \pm 4 \mu \mathrm{~m}$ large (uncertainty provided by the manufacturer). For the $f=150$ tube lens, I was expecting a magnification of $M_{e x, 150}=15$, and measurements give me two values:

$$
\begin{align*}
M_{p h, 150} & =14.6 \pm 0.6 \\
M_{o b, 150} & =14.8 \pm 0.3 \tag{4.22}
\end{align*}
$$

where the error in first case was mainly due to uncertainty of the pinhole dimension whereas in the second case is due to uncertainty in the positions read on the micrometric


Figure 4.8: On the left, a zoom on the tight waist. On the right, data (circles) and fitted profile (solid line). Data to be fitted were obtained by integrating in the wide direction the waist image, after a rotation of $1.15^{\circ}$ due to the CCD chip; the correct angle for the rotation was extracted from the lowest width of the vertically-integrated intensity profile.
screw. The two measurements are compatible, and leading to a magnification of $M_{150}=$ $14.8 \pm 0.3$ through a weighted mean.
The magnification for the $f=300$ was measured only by the use of the obstacle, being this measure more precise: with respect to an expected magnification $M_{e x, 300}=30$, the measured one was $M_{300}=30.4 \pm 0.9$, with same error source as before.
Once having calibrated the magnification, I checked the beam tight waist without the phase plate, to conjugate also the last lens. I found out the waist to be quite insensitive to the translation of the last lens. As a reference, I show the data of the tight beam waist measurement before putting the phase plate: from the Mathematica simulation, it is expected to be $7 \mu \mathrm{~m}$-large, while, as shown in Fig. 4.8, it results to be $9.01 \pm 0.01 \pm 0.02 \mu \mathrm{~m}$ (standard error of the fit and expected systematic error due to the magnification error). I also measured the wide beam waist, as a reference. In this case, as the waist is expected to be about $430 \mu \mathrm{~m}$ large, I didn't use the objective: the image I acquired has thus both the waists, but the tiniest one is only 2-pixels large, so the fitted dimension is not reliable (see Fig. 4.9 right below). The wider waist is $417 \pm 2 \mu \mathrm{~m}$ (standard error of the fit). In this case the fitted waist is more little than the expected one from the Mathematica simulation, which is $430 \mu \mathrm{~m}$ wide. These discrepancies are probably due to the presence in the beam of some contributions from higher modes.

### 4.2.4 Characterization of the $\mathrm{TEM}_{01}$-like profile

I placed the $\pi$ phase plate in the optical path: to align it to be orthogonal to the optical axis, I watched the back-reflection from the surface. At this point, I also checked the final spherical lens to be well orthogonal using the back-reflection, and to be well aligned using a target mounted on the lens and a t-shirt in the cage.


Figure 4.9: On the left, a zoom on the beam waist. On the right, data (circles) and fitted profile (solid line) for wide waist (above) and tight waist (below). Fitted data in both cases were obtained integrating the region of interest on the other direction.

Then, I centred the phase plate: while in the horizontal plane is not so difficult, because the coated region is much bigger than the beam size, in the vertical it was difficult and I used the symmetry of the image acquired by the CCD to try to centre the phase plate as much as possible. The exact position along the optical axis isn't so important because in the direction which the phase plate acts on is collimated.
As in the preliminary measurements, I also observed the intensity profile with and without objective: firstly, without the objective, to get a measure of the wide waist, then adding the objective and the tube lens of the tight $\mathrm{TEM}_{01}$-like profile.


Figure 4.10: Intensity profile of the beam obtained with the $\pi$ phase plate in the focus plane. The plane are the coordinates of the image, while the vertical axis is the intensity (in a.u.).

In Fig. 4.10, I'm showing the intensity profile in the focus plane. From this profile, I fitted the wide waist dimension through integration on the tight direction: the results is

$$
\begin{equation*}
W_{0, x}=442.8 \pm 0.8 \mu \mathrm{~m} \tag{4.23}
\end{equation*}
$$

where the error is the standard error of the fit. The depth of focus of the image was estimated to be about $80 \mu \mathrm{~m}$ by moving the imaging objective. This estimation is also important for our gas: since we expected the Thomas-Fermi radius in the plane to be about $140 \mu \mathrm{~m}$ large, the gas should not feel the modification of the optical potential due to out-of-focus position. In Fig. 4.11 is showed a zoom of the beam in the focus and the fit of the wide direction.
The key measurement of this section is the measure of the waist in the tightly confining direction: I performed measurements both with lower and higher magnification; with the lower magnification I took some points moving the objective along the optical axis, to characterize the profile also slightly out of focus, whereas with the higher magnification I took some points to study the stability of the beam for long times, in the typical operating conditions in the laboratory.


Figure 4.11: Above, a zoom of the acquired image of the $\mathrm{TEM}_{01}$-like green beam. Below, the data (circles) of beam integrated along the vertical direction and the fit (solid line), used to extract the waist dimension.

| $\#$ | Magnification | Measured waist |
| :---: | :---: | :---: |
| 1 | $14.8 \pm 0.3$ | $7.86 \pm 0.04$ |
| 2 | $14.8 \pm 0.3$ | $7.90 \pm 0.04$ |
| 3 | $30.4 \pm 0.9$ | $7.64 \pm 0.03$ |
| 4 | $30.4 \pm 0.9$ | $7.73 \pm 0.03$ |
| 5 | $30.4 \pm 0.9$ | $7.97 \pm 0.03$ |
| 6 | $30.4 \pm 0.9$ | $7.73 \pm 0.03$ |
| 7 | $30.4 \pm 0.9$ | $7.85 \pm 0.02$ |

Table 4.1: Measurements of the equivalent waist performed with the two tube lens. The errors provided are standard errors of the fit.

One example of the beam profile integrated along the wide waist direction and of the fit is given in Fig. 4.12, as is visible, the fit of the whole intensity profile is affected by the presence of large wings and of a non-zero minimum point in the centre, and overestimates a bit the waist dimension and the intensity. This difference between the ideal profile and


Figure 4.12: On the left, a zoom of the magnified image of the intensity profile obtained through the phase plate with the highest magnification. On the right, data (circles) of the beam profile in the tight direction, with the fit (solid line) to extract the equivalent waist. The data were obtained from an image integrating along the wide waist direction.
the measured one is probably due to the presence of higher modes than $\mathrm{TEM}_{00}$ in the incoming beam, but in the end we decided not to add a spatial filter to path, to maintain the most simple configuration. It will be added if needed to trap the atoms.
The measured equivalent beam waist of our intensity profile is thus, performing a weighted average of the data in Tab. 4.1, is

$$
\begin{equation*}
W_{z, P P}=7.81 \pm 0.01 \pm 0.2 \mu \mathrm{~m} \tag{4.24}
\end{equation*}
$$



Figure 4.13: The result of the measurement of waist dimension as function of shift from the focus. The data are circles, while solid line is the fit.
where the first error is the standard error of the mean, while the second is the systematic error due to magnification uncertainty. We can get another measurement of the equivalent waist dimension by a fit using the harmonic approximated expression of Eq. 4.17), for which is needed a precise measurement of the green beam power ${ }^{2}$ and of the wide waist, together with a higher magnification.
I also performed a measurement of the waist in function of the shift with respect to focus: the first thing I noticed was that, shifting the objective from the focus, the two peaks ratio was changing, such that on one side the most intense peak was the left one, whereas on the other side was the right; to me, this was an effect of a slight misalignment between the objective and the axis beam, but I wasn't able to fix it, so it probably have another cause. The results of this measurement seem to show a behaviour different


Figure 4.14: Stability measurements results for the waist (above) and for the position drift (below). The solid line is a guide which indicates the mean of the data. The shaded region is the $95 \%$ confidence level band.

[^19]

Figure 4.15: Simulated trapping profile for green $\mathrm{TEM}_{01}$-like beam, infrared beam and Feshbach coils confining potential. The scale in the two images is $z: x=1: 100$ and $z: y=1: 100$. The green potential along the optical axis is built in the hypothesis of Gaussian waist evolution, which could be incorrect.
from the usual one of a Gaussian beam: in Fig. 4.13, is shown the fit with function $\tilde{W}_{z}(y)=\tilde{W}_{0 z} \sqrt{1+\left(y / y_{R}\right)^{2}}$, which is not matching the data. This can be due or to the asymmetry of the beam for misalignment or to intensity profile itself, which is not a Hermite-Gauss mode.
The last measurements I performed on this trial optical system was a stability measurement in time; to study together also any temperature-dependent variation, I measured also the temperature on the optical table with thermocouple, but the variation was lower than $0.1^{\circ} \mathrm{C}$, which was my sensitivity. In Fig. 4.14, are reported the data of the stability both for the equivalent waist and for the position: both seem to fluctuate of about $\pm 0.4 \mu \mathrm{~m}$, so the $5 \%$ of the waist dimensions. But the fundamental measurement in this sense needs to be performed when the beam is installed in the experimental set-up, both because there are more steep gradients of temperature and sources of noise and because we can have a stabilized power through a PID controller.
Lastly, in Fig. 4.15, I show the profile in planes $x z$ and $y z$ of the trapping potential, with 2.4 W of green beam with the measured parameters, considering the trapping potential due to Feshbach coils and to infrared Gaussian beam with $300 \mu \mathrm{~m}$ waist.

### 4.3 Towards implementation on the experiment

As last step, I also made some designs required to implement the whole optical quasi-2D trap in the experimental set-up. The first one was the implementation of the optical path for the green $\mathrm{TEM}_{01}$-like beam, compatible with the already set horizontal imaging; the second one was the path to create the vertical infrared confining beam; the third was a modification of the horizontal imaging path to make possible the choice of a higher magnification, in order to be able to watch at the vertical width of the cloud.

Lower breadboard


Figure 4.16: The project for installing the green quasi-2D confining potential on the experiment. The circular mirrors are sending and receiving the beam along the vertical $z$ axis, and the dashed line is the vertical beam path.

### 4.3.1 Green beam on atomic cloud

This optical path is very similar to the one used for the characterization of the beam (see Sect. 4.2.2). The main difference, as showed in Fig. 4.16, is having the path divided over different breadboards, for available space reasons. The last lens of the green path, focusing the beam on the atoms, is also used to make a telescope for the red imaging beam: indeed, the green beam is entering the chamber in the same direction of the horizontal imaging (see Fig. 1.11, in Chap. 22). The two beams are combined using a dichroic mirror. On the other side of the science chamber is going to be placed another dichroic mirror, slightly tilted, to reflect towards a beam dump the green beam without affecting the resolution of the imaging.

### 4.3.2 Infrared beam for in-plane confinement

Fig. 4.17 shows the project for the vertical confining beam, with all the optics in the upper breadboard. The optical path is very simple: after the fibre, there is a shutter and then the beam, through two dichroic mirrors for the MOT and imaging red beams and for the DMD green beam. The beam is slightly collimated exiting from the collimator, such to have the $300 \mu \mathrm{~m}$ waist on the atoms. After the objective, the beam is on the imaging path: the first mirror after the tube lens has very low reflectivity for 1064 nm wavelength, thus we expect the most of the intensity is transmitted by the mirror and


Figure 4.17: The project to add the vertical infrared beam (brown lines) and the actual z-MOT and vertical imaging (red lines). The removable quarter-wave plate is needed for the MOT, to have circular polarization, but removed during the imaging routine, which needs a linear polarization. The photodiode, measuring the intensity of the first mirror leak, is going to be used for the PID controller, while the first PBS is used only to clean the beam polarization. For sake of clarity, some waveplates aren't showed.


Figure 4.18: The project for the new horizontal imaging path: the added part is bordered in red. The two flippable mirror are replacing two fixed ones.
damped after it. A bit amount of power is expected to go to the second mirror and to be damped after it.

### 4.3.3 Horizontal imaging

In Fig. 4.18 is showed the new horizontal absorption imaging path: the two flippable mirrors are replacing two fixed others already present on the optical table; this to make possible the choice between the two lower and higher magnification paths. The longer path has two more lenses with focuses $f_{1}=100 \mathrm{~mm}$ and $f_{2}=180 \mathrm{~mm}$, to perform an expected magnification $M=1.8$, additional to the $M=3.77 \pm 0.05$ already present on the imaging system. The two lenses are not conjugated, as we are not going to reconstruct the phase profile but only the intensity; the available space indeed doesn't allow us to have them conjugated, as we want to modify the least possible the imaging system.

## Conclusions and outlooks

In this thesis work, I have demonstrated the possibility to manipulate locally the phase of particle of a ${ }^{6} \mathrm{Li}$ quantum gas by illuminating it with short blue-detuned pulses. I have shown that it's feasible to modify the global phase of a superfluid, imprinting a constant intensity profile, thanks to interference measurements performed in the mBEC gas. Furthermore, using spatially non-uniform shapes from the DMD, I have been able to excite both a rigid motion of the centre of mass with velocities comparable to the critical velocity of the superfluid and a breathing trap mode of the cloud. Eventually, I have also demonstrated the roughness in the intensity profile to have heating effects on the ultracold quantum gas.
Several further applications are now available to be simulated in our system: a Josephson junction dynamics can be driven by the constant phase imprinting on one reservoir [52]. The gas can be trapped in a toroidal geometry and persistent currents can be triggered by the DMD, to study the Kelvin-Helmoltz instabilities [43]. Transport properties can be studied exciting velocities comparable to the critical one and letting the cloud moving across obstacles. Finally, heating effects from the roughness of the intensity profile can be used to study heat transfer and second sound in a superfluid cloud, which is still a not well understood phenomenon [44].
In the second part of the thesis, I have described the design I engineered of a tight anisotropic potential, to confine $10^{4}$ Lithium fermionic atoms in a quasi-two-dimensional degenerate cloud, and the characterization of the optical potential I performed out of the experiment. Moreover, I have discussed the necessary adaptations of the experimental apparatus to install the new potential, with a tunable in-plane confinement and a magnified transverse imaging.
Thanks to the tools discussed in my thesis, we are now planning to study for the first time a point-like disordered potential imprinted on a ultracold atomic gas, instead of the more commonly used speckle disorder. The difference between these two potentials is highlighted in Fig. 4.19 The point-like disorder resembles more impurities in real materials and admits regimes in which classical localization is avoided, because the percolation threshold is close to zero i.e. a classical particle could diffuse through the disorder. The absence of classical localization could make possible the occurrence of the Anderson localization in a 2 -dimensional non-interacting quantum gas, which has never been observed before.
Furthermore, thanks to the broad Feshbach resonance of ${ }^{6} \mathrm{Li}$, our system gives us the op-


Figure 4.19: Images of point-like and speckles disorder. On the left, point-like disorder made by 4 -mirrors wide impurities with density 0.1 ; on the right, speckles disorder obtained from a 1 -mirror wide impurities with density 0.4 by closing the iris.
portunity to study superfluid transport properties tuning the interactions between atoms through the BEC-BCS crossover. The understanding of systems of this kind can contribute to the development of a theoretical model for high- $T_{c}$ granular superconductors, which is still under debate.

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Ci sono poi tutti i familiari che in questi anni sono stati di grande compagnia: zii e cugini che hanno stupito per curiosità e interesse non scontati verso di me. Non posso non citare i nonni Marisa, Luisa e Gigi, perché sono volti che sono un continuo stimolo per me, per il loro affetto e la loro stima. E anche il nonno Pino, che pure se è venuto a mancare prima che io cominciassi questo viaggio, ha riempito questi anni con la sua presenza, perché il cammino è cominciato proprio grazie a lui. Poi ci sono i fratelli: quelli acquisiti, Ben e Paolo, nell'ospitalità a casa nostra, perché sono stati proprio quella forza di presenza e di richiamo quotidiano che sono i proprio fratelli di sangue. Questi ultimi poi, hanno un posto particolare: mio fratello Matteo, che è diventato ancora più caro sposandosi e andando a vivere lontano, e ha dato e dà così tanto alla mia vita, insieme a sua moglie Myriam e negli ultimi mesi a Caterina, che nel suo miracoloso esistere e dipendere in tutto mi ha ricordato che proprio tutto è un dono, anche respirare; mia sorella Francesca, che con tutte le sue domande e le sue certezze mi ha edificato non
poco, e iniziando a diventare donna è riuscita a trascinare tutti in un cambiamento entusiasmante, che spero possa continuare a lungo.
Per la logica con cui sono nati questi ringraziamenti, non possono che finire con coloro senza cui non ci sarebbe stato inizio di questa mia storia grande, perché Uno ha voluto farla grande: i miei genitori. Senza di voi, io non ci sarei: questa è la prima gratitudine e la prima commozione, e basterebbe questo a darvi un valore infinito per me; ma ancora di più, nella fatica, nel dolore e nella gioia siete stati testimoni della presenza grande che avete incontrato nella vita e quindi mi avete fatto un dono più grande dell'esistenza stessa: il dono del segreto del Senso dell'esistenza, e di un luogo una compagnia dove cercarLo. E allora, tutti questi volti, citati e non, che hanno attraversato la mia vita e l'hanno cambiata e fatta crescere, si riconducono a unità in questo, pieni di valore infinito.
Grazie di cuore a tutti, perché ci siete!


[^0]:    ${ }^{1}$ Here, we are not considering stimulated emission process, because we are going to discuss offresonant driving.

[^1]:    ${ }^{2}$ This is true only if the frequency of the field is much closer to that of a particular transition than to those of different transitions. If the transition is not closed, to use the radiative force efficiently one needs to add repumping light.

[^2]:    ${ }^{3}$ I'm not discussing the case of circular polarized light, which makes different shift for the different $m_{J}$ levels, because we are using only linearly polarized traps.

[^3]:    ${ }^{4}$ The phase indeed is unobservable, and all the quantities physically relevant depend on the modulus of the wavefunction.

[^4]:    ${ }^{5}$ Three particles are required because the third particle carries out the excess of energy and momentum freed by the occurrence of a bond.

[^5]:    ${ }^{6}$ The two-body inelastic can be suppressed via the choice of the colliding states, and having a low inelastic scattering rate is very important, to avoid losses due to state modification of the atoms.

[^6]:    ${ }^{7}$ Notice that during the scattering process and after it, the energy $E$ is the same of the incident free particle, because we are dealing with elastic scattering.
    ${ }^{8}$ The short range is needed to define what means "away from the target region".
    ${ }^{9}$ It's evident from the definition of $L=\mathbf{r} \times \mathbf{p}$ that also classically the angular momentum along the propagation axis must vanish.

[^7]:    ${ }^{10}$ The kinetic energy is neglected with respect to the interaction energy: this is valid for a condensate whose size, due to inter-atomic repulsion grows well beyond the extension of the single-particle wavefunction.

[^8]:    ${ }^{1}$ An affine transformation is a linear map between affine spaces (of matrices, in our case) that preserves collinearity (i.e. all points lying on the same line initially still lie on a line after the transformation)

[^9]:    and ratios of distances (e.g. the midpoint of a segment line remains midpoint after transformation).
    ${ }^{2}$ Both this and the previous cited function come from the OpenCV Python module.

[^10]:    ${ }^{3}$ Actually, this is true until the iris is not filtering spatial frequencies comparable with the whole intensity profile

[^11]:    ${ }^{4}$ In Binary mode, state 1 and state 0 correspond to $\pm 12^{\circ}$, while the rest state corresponds to an angle of the single mirror equal to zero.

[^12]:    ${ }^{5}$ The distance at which the object can be moved out of the real focal plane without having it out of focus: $d o f=\frac{\lambda}{N A^{2}}$ in air. For low magnification systems, it depends also from the resolution of the imaging system. |33
    ${ }^{6}$ The depth of focus is the same concept of depth of field but in the conjugated plane.

[^13]:    ${ }^{1}$ This is true if the time is short compared with typical timescales of the excitations driven by $x$, considering the case of perturbation $V(x)=\beta x$; for a trapped gas, this timescales correspond to the inverse of trapping frequencies in the $x$ direction.

[^14]:    ${ }^{2}$ Any subscript is removed because, in the stationary condition, the one due to the perturbing potential is the only current present in the system.
    ${ }^{3}$ Rigid oscillation of the gas around the trap center at the trap frequency $\omega_{x}$.
    ${ }^{4}$ Harmonic oscillation of the cloud width without center of mass motion.

[^15]:    ${ }^{5}$ The superfluid phase, indeed, which is strongly connected to the Bose-Einstein condensation, is rigid against phase twists.
    ${ }^{6}$ Such that the separation between the two condensates becomes negligible with respect to their width.

[^16]:    ${ }^{7}$ The power is calibrated with a photodiode before the DMD. The power is not the real power of the beam illuminating the atoms.

[^17]:    ${ }^{8}$ See Sect. 3.2 for more details about the meaning of this number.

[^18]:    ${ }^{1}$ The scattering length for a tightly confined system can be computed starting from the 3D scattering length, and calculations lead to an effective 2D scattering length for the quasi-2D gas. See 46 for details.

[^19]:    ${ }^{2}$ In my case, as I'm going to explain when talking about the time stability, this measurement couldn't be performed easily because the lack of power stability without a PID controller, due mainly to a malfunction of the Verdi laser.

